Algebra 1

**Structures of Expressions**

**Unit Summary:** In this unit, you will identify expressions and equations based on their structures. You will learn the names of the different parts of an expression or equation. You will also learn about the different number properties that allow you to simplify the structure of an expression, so it is easier to solve.

**GeoGebra Math Practice Tool:** Math Practice is a tool for mastering algebraic notation. It supports students in their step-by-step math work, let's them explore different solution paths, and helps build confidence, fluency, and understanding.[*Teacher Guide*](https://help.geogebra.org/hc/en-us/articles/15294353125533-Teachers-Using-GeoGebra-Math-Practice-in-class) *|* [*Student Guide*](https://help.geogebra.org/hc/en-us/articles/15294377044381-Students-Learn-with-GeoGebra-Math-Practice) *|* [*Video Demo*](https://youtu.be/Injz3kiRx8g)

**Lesson 2 – Parts of Algebraic Expressions**

**Key Words:**

* **coefficient** – a number used to multiply a variable
* **constant** – a quantity having a fixed value that does not change or vary, such as a number
* **equation** – a usually formal statement of the equality or equivalence of mathematical or logical expressions
* **expression** – numbers, symbols and operators grouped together that show the value of something
* **factor** – any of the numbers or symbols in mathematics that, when multiplied together, form a product
* **term** – is either a single number or variable, or numbers and variables multiplied together
* **variable** – a quantity that may assume any one of a set of values, typically represented by a letter

**Objective 1:** In this section, you willinterpret the parts of an algebraic expression in terms of their context.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* A mathematical sentence, also known as an *equation*, can be broken down into parts.



* When you work with math problems, it is important that you know how to identify the different parts of an *equation*, so that you can perform the correct operations.

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| **Parts of an Equation** |
| **Equations*** *Equations* are a formal statement of the equality or equivalence of mathematical or logical expressions
 | This is an *equation*. |
| **Expressions*** *Expressions* are numbers, symbols and operators grouped together that show the value of something
* *Equations* are made up of two *expressions* that are equal
* Each side of the equal sign is an *expression*
 | There are two *expressions* in this *equation*: $$2x−5 $$and $$13 $$. |
| **Terms*** *Expressions* are composed of *terms* which can be separated by a plus or minus sign
* *Terms* are either a single number or variable, or numbers and variables multiplied together
 | The expression $$2x−5 $$is made up of two terms: $$2x $$and $$−5 $$.The expression $$13 $$is its own *term*. |
| **Variables, Constants, and Coefficients*** *Terms* can be broken down into *variables*, *constants*, and *coefficients*
* *Variables* are represented with letters
* *Coefficients* are numbers used to multiply *variables* and can be found immediately in front of a *variable*
* *Constants* have a fixed value that does not change, such as a number that is not accompanied by a *variable*
 | The *term* $$2x $$is made up of a *coefficient*, 2, and a *variable*, $$x $$.The *terms* –5 and 13 are fixed numbers that do not change and are therefore *constant*. |
| **Factors*** *Terms* that multiply together to form a product.
* All *terms* will be composed of two or more *factors* but one of the *factors* could be the “invisible "one that isn’t typically written.
 | In the term $$2x $$, 2 is multiplied with $$x $$, therefore both are factors in $$2x $$. |

* In addition to identifying the parts of an algebraic equation, you must also be alt to interpret the parts’ meaning in the context of word problems.
	+ A *constant* will represent something in the scenario that is fixed and cannot change.
	+ A *variable* will represent an unknown value and is typically denoted with a letter.
	+ A *coefficient* would relate the number of times the problem implies that an unknown value will occur.

**Objective 2:** In this section, you will use context and grouping symbols to interpret parts of an expression as a single entity.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* Recall that a mathematical sentence is made up of many parts (expressions, terms, variables, and coefficients.)
* Grouping symbols are often used in mathematical sentences to show the order in which operations should take place.
	+ () *parentheses*
	+ [] *brackets*
	+ {} *braces*
* The appearance and shape of grouping symbols is different so that it is easy to distinguish the interior groupings you solve first from the exterior one you will solve next.
* In any math problem involving grouping symbols, you will **start with the innermost set**.
	+ Reduce or simplify the math *expression* inside of the grouping symbols.
	+ Once you reduce the *expression* to a single number, you no longer need the grouping symbols.
* Any number before the grouping symbol implies The Distributive Property must be used.
	+ For example, $$5\left(x−3\right)$$

	, tells us that everything inside of the *parentheses* needs to be multiplied by 5.

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| **Example**: Simplify the expression, $$7\left[2^{2}−3\left(5−4\right)\right]$$ |
| **Step 1**: Start with the innermost grouping symbols and simplify.  | The innermost grouping is (5-4) and should be simplified first.$$7\left[2^{2}−3\left(1\right)\right]$$ |
| **Step 2**: Continue to simplify until the parentheses are gone. | The 3 outside of the parentheses indicate multiplication. Simplified Expression: $$7\left[2^{2}−3\right]$$ |
| **Step 3**: Move to the next set of grouping symbols and simplify until the grouping symbols are gone. | The next grouping is grouped with brackets $$\left[2^{2}−3\right]$$, and can be simplified to:$$\left[2^{2}−3\right]=\left[4−3\right]=1$$Simplified Expression: $$7\left[1\right]$$ |
| **Step 4**: Simplify until the grouping symbols are gone. | The 7 outside of the symbols indicate multiplication.$$7\left[1\right]=7$$ |
| **Step 5**: State the answer. | The expression $$7\left[2^{2}−3\left(5−4\right)\right]$$ can be simplified to 7. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Using the equation $C=18+4xC=18+4x$, which of the following statements could be a correct interpretation of the coefficient of a term?Statement #1: Tatiana spent $4.00 per book at the public library.Statement #2: Tatiana spent $18.00 per book at the public library.Statement #\_\_ could be a correct interpretation of the coefficient of a term. | 1 |
| P 2 | Which of the following statements about the equation $2x-5=8+x2x-5=8+x$ is correct?Statement #1: There are two constants in the equation.Statement #2: *x* does not have a coefficient.Statement #3: There are two terms in the equation.Statement #\_\_\_ is correct. | 1 |
| P 3 | Which of the following options contains two coefficients and three terms?Option #1: $2x-3=4-y2x-3=4-y$Option #2: $6+7y=56+7y=5$Option #3: $9x=1-8x9x=1-8x$Option #\_\_\_ contains two coefficients and three terms. | 3 |
| P 4 | Consider the expression $\frac{\left[15+\left(-5\right)\right]+6}{2}$. What expression results from the first step using the order of operations? | $$\frac{10+6}{2}$$ |
| P 5 | Consider the following expression: $\frac{4+(-6)∙2}{-(-3)^{2}}$. What is the simplified denominator? | -9 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Julio and Ashley went to the drive-in theater. They paid an entry fee for the car and individual entry fees for themselves. Their total cost can be modeled by the expression $2.5x+10$. Which of the following answer choices accurately interprets a part of the algebraic expression?  | The entry fee for the car was $10.00. |
| Q 2 |  Given the equation $8x-3y=2x+9y$, which of the following statements is correct? | 8, −3, 2, and 9 are coefficients. |
| Q 3 | An equation can be broken down into expressions, terms, variables, constants, and coefficients. In the equation $0.4-7x=3$, what is/are the constant(s) in the equation? | 0.4 and 3 |
| Q 4 | Use the grouping symbols to interpret the following equation: $\frac{x}{8}=\frac{3(x+4)}{2}$. Which expression in the equation represents a product? | $$3(x+4)$$ |
| Q 5 | Consider the expression $[3\left(4^{2}+32÷4-5\right)]$. Which quotient must be found to simplify the expression? | $$32÷4$$ |

**Lesson 3 – The Commutative Property**

**Key Words:**

* **brace** – one of two marks {} used to connect words or items to be considered together
* **bracket** – one of a pair of marks [ ] used in writing and printing to enclose matter or in mathematics and logic as signs of aggregation
* **equation** – a usually formal statement of the equality or equivalence of mathematical or logical expressions
* **expression** – numbers, symbols and operators grouped together that show the value of something
* **parenthesis** – one or both of the curved marks () used in writing and printing to enclose a parenthetical expression or to group a symbolic unit in a logical or mathematical expression

**Formulas:**

* Commutative Property of Addition: $$a+b=b+a  $$
* Commutative Property of Multiplication: $$a⋅b=b⋅a  $$

**Objective 1:** In this section, you will use the Commutative Property to rewrite algebraic expressions.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas**:

* There are also different rules for numbers, or properties, which are always true no matter what type of expression you are given.

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| **The Commutative Property of Addition** |
| The Commutative Property of Addition states that changing the order in which you add numbers does not change the sum.$$a+b=b+a  $$For example, the expression $$2x+3x+9x $$can also be rewritten as:$$3x+2x+9x $$ **or** $$9x+2x+3x $$\*or any combination of the three terms |

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| **The Commutative Property of Multiplication** |
| The Commutative Property of Multiplication states that changing the order in which you multiply factors does not change the product.$$a⋅b=b⋅a $$For example, the expression $$3⋅2y⋅4x $$can also be rewritten as:$$3⋅4x⋅2y $$ **or** $$4x⋅2y⋅3 $$\*or any combination of the terms |

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| **Example:** Use the Commutative Property of Addition to rewrite the algebraic expression $$5\left(7x+4y\right)$$. |
| **Step 1:** Identify the expression with subtraction or addition.  | The expression $$7x+4y $$can be rewritten as $$4y+7x $$.  |
| **Step 2:** Rewrite the full expression. | Rewritten expression: $$5\left(4y+7x\right)$$ |

**Objective 2:** In this section, you will use the Commutative Property to prove algebraic expressions are equivalent.

*Mathematical Practice Standard: Use the structure of an expression to identify ways to rewrite it.*

**Big Ideas:**

* [Recall](#Bookmark2) the Commutative Property of Addition and Multiplication.
* *The Commutative Property* and can prove that two algebraic expressions that appear differently are equivalent.
* Two algebraic expressions are equivalent if the terms and numbers can be written in a new order and the expressions are equivalent.

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| **Example:** Prove that these algebraic expressions are equivalent.Expression 1: $$3\left(2x+1\right)$$Expression 2: $$\left(2x+1\right)3$$ |
| The Commutative Property of Multiplication states that the order in which numbers are multiplied does not change the answer.In this example, each expression has two terms: $$\left(2x+1\right)$$ and 3. The order in which they are multiplied will result in the same answer. Expression 1: $$3\left(2x+1\right)=6x+3$$Expression 2: $$\left(2x+1\right)3=6x+3$$ |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Use the Commutative Property of Addition to rewrite the algebraic expression: $4x^{2}+8x$. | $$8x+4x^{2}$$ |
| P 2 | Use the Commutative Property of Addition to rewrite the algebraic expression: $ab^{2}+c^{3}$. | $$c^{3}+ab^{2}$$ |
| P 3 | Use the Commutative Property of Multiplication to rewrite the algebraic expression: $a^{20}⋅x^{3}$. | $$x^{3}∙a^{20}$$ |
| P 4 | Using the Commutative Property, fill in the blanks so that the two algebraic expressions are equivalent.(14)(\_\_\_)(2)=(7)(\_\_\_)(14) | 7; 2 |
| P 5 | Using the Commutative Property, fill in the blanks so that the two algebraic expressions are equivalent.$$5+\\_\\_\\_+1+7=4+\\_\\_\\_+7+1$$ | 4; 5 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
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| Q 1 | Which of the following correctly demonstrates the use of the Commutative Property of Multiplication? | $$2\left(b^{10}+z^{11}\right)=(b^{10}+z^{11})∙2$$ |
| Q 2 | Which of the following correctly demonstrates the Commutative Property of Addition? | $$abc+xyz=xyz+abc$$ |
| Q 3 | Use the Commutative Property to determine the missing step in proving the equivalence of $12a+10+a= =10+a+12a$.Step 1: [missing]Step 2: $10+13a=10+13a$Step 3: The expressions are equivalent because they both equal the same value. | $$10+12a+a=10+13a$$ |
| Q 4 | Substituting 1 for x in the equation $$\left(x⋅3\right)5=5\left(x⋅3\right)$$ is a test case for which property? | The Commutative Property of Multiplication |
| Q 5 | Which step contains an error using the Commutative Property of Addition in verifying $8+x^{2}+17x-x^{2}+4=10+4x^{2}+10x-4x^{2}+2+7x$?Step 1: $8+4+x^{2}-x^{2}+17x=10+2+10x-4x^{2}+7x +4x^{2}$Step 2: $8+4+17x+x^{2}-x^{2}=10+2+4x^{2}-4x^{2}+10x+7x$Step 3: $12+17x+x^{2}=12-x^{2}+17x$ | Step 3 |

**Lesson 4 – The Associative Property**

**Key Words:**

* **Associative Property** – a rule stating that the way factors are grouped in an addition problem or in a multiplication problem has no effect on the product
* **equivalent** – equal in force, amount, or value; having the same solution set (equivalent equations)
* **expression** – numbers, symbols and operators grouped together that show the value of something

**Formulas:**

* Associative Property of Addition: $$a+\left(b+c\right)=\left(a+b\right)+c$$
* Associative Property of Multiplication: $$a⋅\left(b⋅c\right)=\left(a⋅b\right)⋅c$$

**Objective 1:** In this section, you will use the Associative Property to rewrite algebraic expressions.

*Mathematical Practice Standard: Use the structure of an expression to identify ways to rewrite it.*

**Big Ideas**:

* Recall that mathematics has different properties for numbers that hold true for every type of algebraic *expression*.
* [Recall](#Bookmark2) the Commutative Property of Addition and Multiplication.
* *The Associative Property* is another property that **holds true for addition and for multiplication only**, never for subtraction or division.

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| **The Associative Property** |
| The Associative Property states that the groupings of numbers in an addition or in a multiplication problem can be changed without affecting or changing the result of that problem. Addition: $$a+\left(b+c\right)=\left(a+b\right)+c$$Multiplication: $$a⋅\left(b⋅c\right)=\left(a⋅b\right)⋅c$$ |

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| **Example:** Use the Associative Property of Multiplication to rewrite the following expression: $$7a⋅\left(8b⋅2\right)$$ |
| * In this expression, three numbers are being multiplied together. The numbers inside the parentheses are $$8b $$and 2.
* The expression can be rewritten equivalently by changing the grouping. For example, it can be rewritten in the following ways:
	+ $$\left(7a⋅8b\right)⋅2$$ or $$\left(7a⋅2\right)⋅8b$$
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**Objective 2:** In this section, you will use the Associative Property to prove algebraic expressions are equivalent.

*Mathematical Practice Standard: Use the structure of an expression to identify ways to rewrite it.*

**Big Ideas:**

* [*The Associative Property*](#Bookmark3)can prove that two algebraic *expressions* that appear different are *equivalent*.
	+ This property can help prove equivalent expressions by allowing you to write parentheses and groupings of an addition or multiplication problem in a new order.

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| **Example**: Use the Associative Property of Addition to prove that these algebraic expressions are equivalent.Expression 1: $$x+\left(4x+5\right)$$Expression 2: $$5+\left(x+4x\right)$$ |
| **Step 1:** Write the first expression in its simplest form. | $$x+\left(4x+5\right) = x+4x+5 = 5x+5$$Expression 1: $$ 5x+5 $$ |
| **Step 2**: Write the second expression in its simplest form. | $$5+\left(x+4x\right) = 5+x+4x = 5+5x$$Expression 2: $$5+5x $$ |
| **Step 3:** Use the Commutative Property to write the terms of expression 2 in a different order. | $$5+5x = 5x+5 $$Expression 2: $$5x+5 $$ |
| **Step 4:** State the answer. | $$x+\left(4x+5\right)$$ and $$5+\left(x+4x\right)$$ can be rewritten as $$5x+5 $$. Therefore, both algebraic expressions are equivalent. |

 **Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Use the Associative Property of Addition to rewrite the expression $(7+8)+6$ as an equivalent expression. | $$7+(8+6)$$ |
| P 2 | Rewrite the expression $3x+(2y+z)$ using the Associative Property of Addition. | $$\left(3x+2y\right)+z$$ |
| P 3 | Apply the Associative Property of Multiplication to rewrite the expression $p⋅(q⋅r)$. | $$(p∙q)∙r$$ |
| P 4 | Use the Associative Property to rewrite the expression $3x+(x+2)$ by combining like terms. | $$4x+2$$ |
| P 5 | Use the Associative Property to rewrite the expression $$9x+\left(2x+3\right)$$ by combining like terms.  | $$11x+3 $$ |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Which of the following expressions could you rewrite using the Associative Property? | $$(9∙5)∙(8∙3)$$ |
| Q 2 | Which of the following correctly demonstrates the Associative Property of Addition? | $$\left(x+y\right)+z+r=x+\left(y+z\right)+r$$ |
| Q 3 | Which of the following demonstrates the Associative Property of Multiplication? | $$(3⋅5)⋅2=3⋅(5⋅2)$$ |
| Q 4 | Use the Associative Property to determine which expressions are equivalent. | $$-3⋅(4x⋅-2)⋅-6y=(-3⋅4x)(-2⋅-6y)$$ |
| Q 5 | According to the Associative Property, which expression is equivalent to $30m+(21m-53)+(18-2m)$? | $$51m+(-53+18)-2m$$ |

**Lesson 5 – The Distributive Property**

**Key Words:**

* **binomial** – an algebraic expression with two terms
* **Distributive Property** – a property of algebra that states that multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together; $a(b+c)=ab+ac$
* **equivalent** – equal in force, amount, or value; having the same solution set (equivalent equations)
* **factor** – any of the numbers or symbols in mathematics that, when multiplied together, form a product
* **polynomial** – an algebraic expression composed of variables, constants, coefficients, operators (such as addition and subtraction), and whole number exponents

**Formulas:**

* The Distributive Property: $$a\left(b+c\right)=ab+ac$$

 and $$a\left(b−c\right)=ab−ac$$
* Difference of Squares: $$a^{2}−b^{2}=\left(a+b\right)\left(a−b\right)$$

**Objective 1:** In this section, you will use the Distributive Property to rewrite algebraic expressions.

*Mathematical Practice Standard: Use the structure of an expression to identify ways to rewrite it.*

**Big Ideas**:

* Recall that mathematics has different properties for numbers that hold true for every type of algebraic *expression*.
	+ [Recall](#Bookmark2) the Commutative Property of Addition and Multiplication.
	+ [Recall](#Bookmark3) the Associative Property.
* *The Distributive Property* is another property:

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| **The Distributive Property** |
| The Distributive Property states that multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together.$$a\left(b+c\right)=ab+ac$$$$a\left(b−c\right)=ab−ac$$ |

* The *Distributive Property* can be used to multiply two *binomials* together to create a *polynomial*.
	+ A *binomial* is an algebraic expression with two terms.
	+ A *polynomial* is an algebraic expression that consists of a sum of terms - composed of variables, constants, coefficients, operators (addition and subtraction), and whole number exponents.
* One way to complete the*Distributive Property* is to use the tabular method.
	+ The tabular model involves creating a table with rows and columns representing the terms of the *polynomial*.
	+ The more terms an expression has, the larger the table will be.
	+ By multiplying the values in each cell and combining like terms, you can calculate the final *expression*.

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| **Example:** Use the Distributive Property to rewrite the following expression using the tabular model.$$\left(x+4\right)\left(x−5\right)$$ |
| **Step 1:** Set up a table to multiply the binomials.

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|  | $$x $$ | $$−5 $$ |
| $$x $$ |  |  |
| $$+4 $$ |  |  |

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| **Step 2:** Fill the remaining cells with terms you have placed in the outer row and column, writing a multiplication sign between them.  |
| **Step 3:** Multiply the expressions in each cell with two terms.  |
| **Step 4:** Collect each term that you calculated in the table and write them horizontally to create a polynomial. $$x^{2}−5x+4x−20$$ |
| **Step 5:** Combine like terms.$$x^{2}−x−20$$ |
| **Step 6:** State the answer:Applying the Distributive Property using the tabular model to the expression $$\left(x+4\right)\left(x−5\right)$$ results in the equivalent expression $$x^{2}−x−20$$. |

**Objective 2:** In this section, you will use the Distributive Property to prove algebraic expressions to be equivalent.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

* [*The Distributive Property*](#Bookmark4) can be used to prove that two algebraic expressions are *equivalent* by multiplying two *expressions* together, then simplifying by combining like terms. If the simplified result is the same as another *expression*, then the two *expressions* are *equivalent*.
	+ [Recall](#Bookmark5) the tabular method for multiplying two binomials.

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| **Example**: Use the Distributive Property to prove that these algebraic expressions are equivalent.Expression 1: $$\left(3x+1\right)\left(x−4\right)$$Expression 2: $$2x^{2}−10x−x+x^{2}−4$$ |
| **Step 1:** Rewrite expression 1 using the Distributive Property. \* [Recall](#Bookmark5) that you can use the tabular method to multiply the two binomials. | Expression 1: $$\left(3x+1\right)$$ $$\left(x−4\right)$$

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|  | $$x $$ | $$−4 $$ |
| $$3x $$ | $$x⋅3x=3x^{2}$$ | $$−4⋅3x=−12x $$ |
| $$1 $$ | $$x⋅1=x $$ | $$−4⋅1=−4 $$ |

Put the calculated terms from the table together and combine like terms: $$3x^{2}−12x+x−4$$$$3x^{2}−11x−4$$**Expression 1:** $$3x^{2}−11x−4$$ |
| **Step 2:** Simplify expression 2 by combining like terms and adding coefficients. | Expression 2: $$2x^{2}−10x−x+x^{2}−4$$Combine like terms with $$x^{2}$$: $$3x^{2}−10x−x−4$$Combine like terms with *x*: $$3x^{2}−11x−4$$**Expression 2:** $$3x^{2}−11x−4$$ |
| **Step 3:** State the answer. | Using the Distributive Property and simplifying by combining like terms shows that both expressions given can be rewritten as $$3x^{2}−11x−4$$. |

* The difference of squares is a specific *polynomial* that has its own rule for factoring.

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| **The Difference of Squares** |
| The difference of squares is a specific polynomial where one squared term is subtracted from another squared term. Difference of Squares Factoring Rule: $$a^{2}−b^{2}=\left(a+b\right)\left(a−b\right)$$For example, $$x^{2}−9$$ is the difference of $$x^{2}$$ minus $$3^{2}$$ and can thus be written as $$x^{2}−3^{2}=\left(x+3\right)\left(x−3\right)$$. |

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| **Example:** Use the Distributive Property to prove that these algebraic expressions involving difference of squares are equivalent.Expression 1: $$2z^{2}−z^{2}−49$$Expression 2: $$\left(z−7\right)\left(z+7\right)$$ |
| **Step 1:** Rewrite expression 1 by combining like terms. | Expression 1: $$2z^{2}−z^{2}−49$$**Rewritten Expression 1:** $$z^{2}−49$$ |
| **Step 2:** Rewrite expression 2 using the properties for the difference of squares. | Difference of Squares Rule: $$a^{2}−b^{2}=\left(a+b\right)\left(a−b\right)$$Expression 2: $$\left(z−7\right)\left(z+7\right)$$$$\left(z−7\right)\left(z+7\right)=z^{2}−7^{2}=z^{2}−49$$**Rewritten Expression 2:** $$z^{2}−49$$ |
| **Step 3:** State the answer:  | Both expressions given are equivalent to $$z^{2}−49$$ and are therefore equivalent expressions. This equivalency is an example of the difference of squares. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Use the Distributive Property to rewrite the polynomial $$\left(x+8\right)\left(2x−2\right)$$\_\_\_$x^{2}+\\_\\_\\_x+\\_\\_\\_$ | 2; 4; -16 |
| P 2 | Rewrite the expression $3x(x + 3)$ using the Distributive Property. | $$3x^{2}+9x$$ |
| P 3 | Find the product of the polynomials $(2x+1)(x-4)$.\_\_\_$x^{2}+\\_\\_\\_x+\\_\\_\\_$ | 2; -7; -4 |
| P 4 | From first to last, order the steps to prove that expression A is equivalent to expression B.Expression A: $(x+4)(x-2)$Expression B: $x^{2}+2x-8$Option #1: $(x+4)(x-2)$(Start)Option #2: $x^{2}-2x+4x-8$Option #3: $x^{2}+2x-8$Option #4: $x(x)-x(2)+4(x)-4(2)$ | 1; 4; 2; 3 |
| P 5 | Identify two expressions that will be equivalent to $2x^{2}-8x-10$ when distributed. Enter the option with the lesser number first.Option #1: $2x(x-4x-5)$Option #2: $(2x+2)(x-5)$Option #3: $2x(x-5)+2(x-5)$Option #4: $(x-5)(x-3)$Option #5: $-6x-10$ | 2; 3 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | According to the Distributive Property, which is a way to rewrite the algebraic expression $(3x-1)(x-4)$? | $$3x(x)+3x(-4)+-1(x)+-1(-4)$$ |
| Q 2 | Use the Distributive Property to find which expression is equivalent to $9x^{2}-25$. | $$(3x+5)(3x-5)$$ |
| Q 3 | Use the Distributive Property to verify which expression is equivalent to $(x-5)(x-3).$ | $$x^{2}-8x+15$$ |
| Q 4 | Which expression is equivalent to $(z+4)(z-4)$? | $$z^{2}+4z-4z-16$$ |
| Q 5 | What is another way to write the expression $2(x+1)(x+2)$? | $$2x^{2}+6x+4$$ |

**Lesson 6 – Adding & Subtracting Polynomials**

**Key Words:**

* **closed system** – an operation that when performed on members of a set produce another member of the same set
* **coefficient** – a number used to multiply a variable
* **Commutative Property of Addition** – the property of operations which states that changing the order in which you add numbers does not change the sum; $a+b=b+a$
* **Distributive Property** – a property of algebra that states that multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together; $a(b+c)=ab+ac$
* **expression** – numbers, symbols and operators grouped together that show the value of something
* **like term** – a term whose variables and exponents are the same as those of another term
* **polynomial** – an algebraic expression composed of variables, constants, coefficients, operators (such as addition and subtraction), and whole number exponents
* **term** – is either a single number or variable, or numbers and variables multiplied together

**Formulas:**

* Associative Properties
	+ Addition: $$a+\left(b+c\right)=\left(a+b\right)+c$$
	+ Multiplication: $$a⋅\left(b⋅c\right)=\left(a⋅b\right)⋅c$$
* Commutative Properties
	+ Addition: $$a+b=b+a  $$
	+ Multiplication: $$a⋅b=b⋅a  $$
* Distributive Property: $$a\left(b+c\right)=ab+ac$$

 or $$a\left(b−c\right)=ab−ac$$
* Difference of Squares: $$a^{2}−b^{2}=\left(a+b\right)\left(a−b\right)$$

**Objective 1:** In this section, you will add polynomial expressions.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas**:

* Recall that a *polynomial expression* is composed of variables, constants, coefficients, operators (addition and subtraction, and whole numbers.
* You can add and subtract different *polynomials* to create one *polynomial* using the following guidelines:
	+ 1. Use the [*Distributive Property*](#Bookmark4), if necessary.
	+ 2. Use the [*Commutative Property*](#Bookmark2) to group *like terms*. Arrange with descending powers.
		- Recall that a “power” is what a number or variable is raised to. For example, $$2x^{3}$$

		 has a power of three.
	+ 3. Simplify, by combining *like* terms *coefficients*.
		- *Like terms* are terms that have the **same variable and exponents**.
		- 

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| **Example:** Subtract the polynomials. |
| **Step 1:** Use the Distributive Property, if necessary. | The minus sign between the two polynomials must be applied to each term in the second polynomial $$\left(2x^{2}+4x−6\right)$$.This can be done quickly by changing the sign of each term in the second polynomial:$$−\left(2x^{2}+4x−6\right)=−2x^{2}−4x+6$$New expression:  |
| **Step 2:** Use the Commutative Property to group like terms and arrange in descending power. | Rearrange the expression so that the terms go from the highest power to the lowest power. \*Notice that the term with the highest power of 2 is first, then the term with a power of 1 is next, then the term with no power is last. |
| **Step 3:** Simplify by combining like terms. | There are two terms with the same variable and power of $$x^{2}$$ and there are two constants with now variable and now power, these can be combined:New expression: $$−1x^{2}−4x+2$$\*Notice that $$−4x $$had not like terms in this problem. |
| **Step 4:** State the answer. | $$\left(x^{2}−4\right)−\left(2x^{2}+4x−6\right)=−1x^{2}−4x+2$$ |

**Objective 2:** In this section, you show that polynomials form a closed system under addition and subtraction.

*Mathematical Practice Standard: Construct viable arguments and critique the reasoning of others.*

**Big Ideas:**

* In algebra, a *closed system* under addition and subtraction means that if you have a group of numbers, you can add or subtract any two numbers from that group, and the answer will always be another number in that same group.
	+ Recall when you added and subtracted rational numbers, the result was always another rational number.
* What does this mean for *polynomials*?
	+ Recall that a *polynomial* is defined as an expression with one or more terms, in which each term consists of a variable raised to a nonnegative integer exponent multiplied by a coefficient.
	+ When two *polynomials* are added or subtracted, the resulting expression is still considered a *polynomial*, even when simplified to a single term, a constant, or zero.
* Let’s use an example to demonstrate a *closed system*:

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| **Is the sum a polynomial?** | **Is the difference a polynomial?** |
| The sum of the two expressions is:Yes, the sum meets the definition of a polynomial.  | The difference of the two expressions is:Yes. Zero is a number and an acceptable coefficient so this expression meets the definition of a polynomial. |

**Practice Questions and Answers**

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| --- | --- | --- |
|  | Question | Answer |
| P 1 | Add the polynomial expressions $2r^{2}-3r+4$ and −$2r^{2}+3r+6$. | 10 |
| P 2 | Add the expressions.Expression 1: $-3k+4k^{3}-2$Expression 2: $9k^{2}-3k^{3}+4k-3$$$\\_\\_\\_k^{3}+\\_\\_\\_k^{2}+\\_\\_\\_k+\\_\\_\\_$$ | 1; 9; 1; -5 |
| P 3 | Simplify $(9x^{2}-6x+2)-(x^{2}+4x-39).$The difference of the polynomials is\_\_\_. | $$8x^{2}-10x+41$$ |
| P 4 | Fill in the blanks to complete the polynomial equation that shows the subtraction of the second expression from the first and the resulting difference.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|   | $$9x^{2}$$ | + | *\_\_\_x* | + | 13 |
| – | $$2x^{2}$$ | + | $$12x$$ | + | \_\_\_ |
|   | $$\\_\\_\\_x^{2}$$ | + | $$5x$$ | + | 2 |

 | 17; 11; 7 |
| P 5 | Simplify $(\frac{1}{4}x^{2}-3x+2.25)+(0.75x^{2}+2x-\frac{1}{4}).$The sum of the polynomials is \_\_\_. | $$x^{2}-x+2$$ |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Add the polynomial expressions $3-2p-5p^{2}$ and $p^{4}-3p+4$. | $$p^{4}-5p^{2}-5p+7$$ |
| Q 2 | What is $(3x^{2}-2)+(11-5x)$? | $$3x^{2}-5x+9$$ |
| Q 3 | Which example correctly demonstrates polynomial addition? | $$(2x^{2}+6x+1)+(3x^{2}+3x+9)=5x^{2}+9x+10$$ |
| Q 4 | Which example correctly demonstrates polynomial subtraction? | $$\left(2x^{2}+6x+1\right)-\left(3x^{2}+3x+9\right)=x^{2}+3x-8$$ |
| Q 5 | What does it mean for a set to be "closed"? | An operation performed on members of that set will result in a member of the same set. |

**Lesson 7 – Multiplying Polynomials**

**Key Words:**

* **base** – a number (such as 5 in $5^{6.444}$or $5^{7}$) that is raised to a power
* **binomial** – an algebraic expression with two terms
* **closed system** – an operation that when performed on members of a set produce another member of the same set
* **coefficient** – a number used to multiply a variable
* **Distributive Property** – a property of algebra that states that multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together; $a(b+c)=ab+ac$
* **exponent** – a symbol written above and to the right of a mathematical expression to indicate the operation of raising to a power; represents the number of times a base value is multiplied by itself, and simplifies repeated multiplication by expressing it as a shorter mathematical notation
* **like term** – a term whose variables and exponents are the same as those of another term
* **monomial** – a mathematical expression consisting of a single term
* **polynomial** – an algebraic expression composed of variables, constants, coefficients, operators (such as addition and subtraction), and whole number exponents
* **set** – a collection of elements, especially mathematical ones (such as numbers or points)
* **trinomial** – a mathematical expression consisting of three terms

**Formulas:**

* Distributive Property: $$a\left(b+c\right)=ab+ac$$

 or $$a\left(b−c\right)=ab−ac$$
* Product Property of Exponents: $$b^{m}⋅b^{n}=b^{m+n}$$

**Objective 1:** In this section, you will multiply polynomial expressions.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas**:

* The rules for multiplying *polynomial* expressions are similar to the rules for multiplying numeric expressions.
* The major difference stems from the variables and *exponents* that are present in *polynomial* expressions.
* When multiplying terms that have the same *exponents*, the terms can be combined (or simplified) if they contain the same *base*.
	+ Recall that an exponent's *base* is the number or variables raised to a power.
	+ For *exponents* to be simplified, each expression **must contain exactly the same *base* with the exact same variables**.
	+ The order in which the *variables* appear does NOT affect the ability to simplify the *exponents*.
	+ 
* It’s important to recall what you know of *exponent* rules, specifically when multiplying exponent expressions:
	+ When multiplying *exponent expressions* with the same *base*, the *exponents* must be **added together**.
		- Product Property of Exponents: $$b^{m}⋅b^{n}=b^{m+n}$$
		- For example: $$x^{2}⋅x^{4}=x^{2+4}=x^{6}$$
	+ When multiplying *exponent expressions* with *coefficients*, or a constant factor with a variable, **proceed with multiplication** as usual.
		- For example: $$5x^{3}⋅2x^{6} = 10x^{9}$$
		- Note the difference between coefficient and exponent values. The 5 and 2 coefficients are multiplied while the exponents 3 and 6 are added together.
* Let’s combine what we have learned in this unit so far to multiply polynomial expressions:

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| **Example**: Multiply the polynomials. |
| **Step 1:** Recall the [Distributive Property](#Bookmark4) and [tabular method](#Bookmark5). Set up the table.

|  |  |  |
| --- | --- | --- |
|  | $$2x^{2}$$ | $$3y $$ |
| $$4x $$ | $$2x^{2}⋅4x$$ | $$3y⋅4x $$ |
| $$y^{3}$$ | $$2x^{2}⋅y^{3}$$ | $$3y⋅y^{3}$$ |

 |
| **Step 2:** Use the rules for multiplying exponent expressions to complete the calculations in the four white squares.\*Recall that when multiplying exponent expressions; multiply coefficients, add exponents (if they have the same base.)Notice the calculations for $$3y⋅4x = 12xy $$and $$2x^{2}⋅y^{3}=2x^{2}y^{3}$$. The variables *x* and *y* are not alike and therefore can only be combined by multiplication, not using any exponent rules.  |
| **Step 3:** Collect each term and write them horizontally to create a polynomial. $$8x^{3}+12xy+2x^{2}y^{3}+3y^{4}$$ |
| **Step 4:** Group and combine like terms.There are no like terms in this example, so it stays as is. |
| **Step 5:** Rearrange so that powers are in descending order. In this case, where terms have more than one variable like $$12xy $$and $$2x^{2}y^{3}$$. Combine the powers of each variable to identify its placement. For example, $$2x^{2}y^{3}$$ has a total power of 5, making it the highest and first term in the polynomial.$$2x^{2}y^{3}+3y^{4}+8x^{3}+12xy$$ |
| **Step 6:** State the answer.Applying the Distributive Property to the expression  results in the equivalent expression $$2x^{2}y^{3}+3y^{4}+8x^{3}+12xy$$. |

**Objective 2:** In this section, you will show how multiplying polynomials forms a closed system.

*Mathematical Practice Standard: Construct viable arguments and critique the reasoning of others.*

**Big Ideas:**

* *The Closure Property* states that when performing a certain operation on elements within a *set*, the result will always remain within that same *set*.
* What does this mean for *polynomials*?
	+ Recall that a *polynomial* is defined as an expression with one or more terms, in which each term consists of a variable raised to a **nonnegative integer** **exponent** multiplied by a coefficient.
	+ When two *polynomials* are multiplied, the resulting expression is still considered a *polynomial.*

**Practice Questions and Answers**

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| --- | --- | --- |
|  | Question | Answer |
| P 1 | Multiply the polynomial expression $(x+2)(x-4)$.$$\\_\\_\\_x^{2}+\\_\\_\\_x+\\_\\_\\_$$ | 1; -2; -8 |
| P 2 | Multiply to simplify the polynomial expression $(r-1)(r^{2}-2r+3)$.$$\\_\\_\\_r^{3}+\\_\\_\\_r^{2}+\\_\\_\\_r+\\_\\_\\_$$ | 1; -3; 5; -3 |
| P 3 | Simplify the polynomial expression $(xy-2)(x^{2}+1)$.$$\\_\\_\\_x^{3}y+\\_\\_\\_x^{2}+\\_\\_\\_xy+\\_\\_\\_$$ | 1; -2; 1; -2 |
| P 4 | Which of the following shows that polynomials form a closed system under multiplication? Option #1: $3(\frac{2}{x}+5)=\frac{6}{x}+15$Option #2: $8x+7=5x+3x+1+6$ Option #3: $(2x^{2}-4)(3y+6)=6x^{2}y+12x^{2}-12y-24$Option #4: $2x^{\frac{1}{2}}∙3=6\sqrt{x}$Option #\_\_\_ | 3 |
| P 5 | Which of the following options correctly describes if the following polynomial forms a closed system under multiplication?$$-3(\frac{5}{x}+4y)=-\frac{15}{x}-12y$$Option #1: Yes, because the result of multiplying the polynomials is also a polynomial. Option #2: No, because the exponent of *x* is not a positive integer. Option #3: No, because multiplying the polynomials resulted in subtraction. Option #\_\_\_ | 2 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Multiply the polynomial $(b+8)(3b-6)$ to simplify. | $$3b^{2}+18b-48$$ |
| Q 2 | What is the product of the polynomials $(x^{2}y+2)(x^{2}-y)?$ | $$x^{4}y-x^{2}y^{2}+2x^{2}-2y$$ |
| Q 3 | Which expression is equivalent to $x^{3}(2+y^{5})$? | $$2x^{3}+x^{3}y^{5}$$ |
| Q 4 | Which of the following equations is an example showing that polynomials form a closed system under multiplication? | $$\left(x^{2}+1\right)\left(x−\frac{1}{2}\right)=x^{3}−\frac{1}{2}x^{2}+x−\frac{1}{2}$$ |
| Q 5 | Use multiplication to find the product that demonstrates the Closure Property of multiplication of polynomials.$$(\frac{1}{2}x^{2}-3)(4y^{3}+5x^{2})$$ | $$2x^{2}y^{3}+\frac{5}{2}x^{4}-12y^{3}-15x^{2}$$ |