Algebra 2

**Polynomial Functions & Graphs**

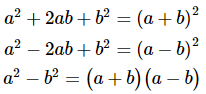
**Unit Summary:** In this unit, you will work with polynomial functions. You'll start by factoring polynomial equations to solve them, then connect the factored form to the graph's properties. You'll create polynomial functions from given properties and values, evaluate them at specific points, graph them, identify patterns, and interpret key points on these graphs.

**Lesson 2 – Roots of Polynomials**

**Key Words:**

* **decompose** – to break down an equation into smaller parts
* **degree** – the value of the highest exponent on any variable in a polynomial
* **linear factor** – a factor in the form ax±c where a and c are constants; the exponent on the variable is 1
* **quadratic function** – a function in the form
* **root –** the value of x that makes an algebraic expression equal to zero
* **x-intercept** – the coordinates (x,0) where the graph touches the x-axis
* **Zero Product Property** – a rule stating that if the product of two or more factors is equal to zero, then at least one of the factors has to be zero
* **zeros of a polynomial** – the input (x) values that make a polynomial have a value of zero

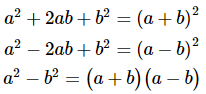
**Formulas:**

* Zero Product Property:If , then, ,or both and equal to zero.
* Common Polynomial Identities:
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**Objective 1:** In this section, you will solve polynomial equations by decomposing polynomials into linear factors.

*Mathematical Practice Standard: Model with mathematics.*

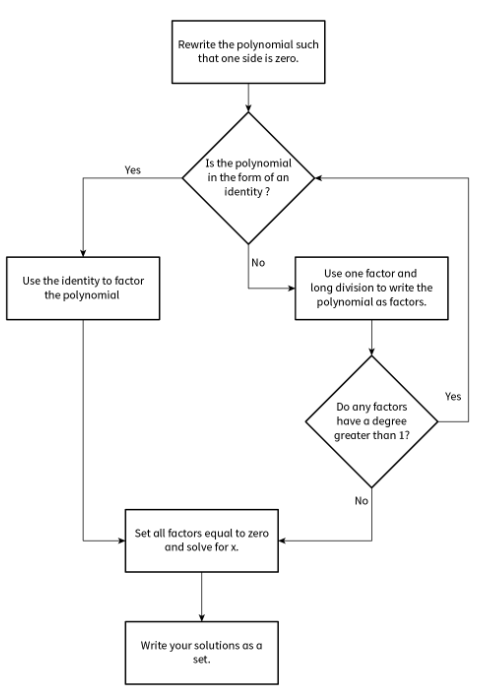
**Big Ideas**:

* Recall that common polynomial identities include the following:
  + 
* The polynomial identities *decompose* the expression into *linear factors* that help you find the *roots*.
  + *Linear factors* are in the form . The variable is raised to a power of one.
  + *Roots* are the value of *x* that makes an algebraic expression equal to zero.
* Factoring is useful when you want to *decompose* a polynomial into its *linear factors* and solve an equation to find its *roots*.
  + 1. Identify the form of the polynomial
  + 2. Apply the *Zero Product Property*

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| **Zero Product Property**  The *Zero Product Property* holds that if the product of two or more factors is zero, then at least one factor must be zero.  If , then , or both and equal to zero. |

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| **Example:** Decompose the polynomial into linear factors and solve the unfactored equation. | |
| **Step 1:** Move all terms over to one side using inverse operations. This puts the polynomial in standard form. | Move the 6 by adding 6 to each side. |
| **Step 2:** Identify the polynomial identity that fits form of the polynomial. |  |
| **Step 3:** Decompose the polynomial using this identity. |  |
| **Step 4:** Write the result as linear factors. |  |
| **Step 5:** Apply the Zero Product Property that states one or more of these factors must be equal to 0. | Since both factors are the same, you only need to solve one equation. |
| **Step 6:** State the answer. | The solution set is . |

* If the polynomial cannot be put in the form of a polynomial identity, you can use polynomial division.
* This flowchart can help you follow the process of *decomposing* polynomials into *linear factors*.



**Objective 2:** In this section, you will connect the zeros of a polynomial function to the *x*-intercepts on its graph.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas:**

* Recall that the process of decomposing a polynomial helped you to solve polynomial equations and find the zeros, or *roots*, of the polynomial.
* The graph of an equation is connected to the zeros of a polynomial.
  + The *zeros of a polynomial* are the input values that make its value zero. Also known as the *roots*.
  + You can connect the linear factors of a polynomial to the *zeros of the polynomial*.
* You can also find the *zeros of a polynomial* by locating the *x-intercepts* on a graph.
  + When the polynomial touches the *x-*axis, the value of the polynomial is equal to zero.
* Let’s use this example to find the roots, or zeros, of the polynomial in two different ways.

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| **Example:** Find the zeros of . | |
| **Algebraically** | **Graphically** |
| To find the zeros, set each factor equal to zero and solve for *x*.  There are three factors:  The values that make the polynomial zero are  The *x*-values are where . | Now consider the graph of .  Notice that the graph touches the x-axis at the same values of .  The zeros are the *x-*values where , that means at those *x*-values, and gives the following coordinates: .  These are the locations of the *zeros*, or *roots* of the polynomial. Another name for these points is the *x-intercept*. |

* You can also use the *zeros of a polynomial*, when given a graph, to find the factors of the polynomial.
  + If you know the *zero*, you can find the factor that it would create in a polynomial.

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| **Example:** The function has zeros at and . Identify the factors that make up the polynomial and write the function. | |
| **Step 1:** Rearrange the equations for the zeros to equal 0. | Frist zero:    Second zero: |
| **Step 2:** Identify the factors. | The resulting equations identify the factors.  are two factors of the polynomial that makes the graph. |
| **Step 3:** Put the factors together and expand it. | When expanded: |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Write the product as linear factors: . |  |
| P 2 | Decompose the polynomial into its linear factors, given that is a factor. |  |
| P 3 | How many intercepts does the function have when graphed?  The function has \_\_\_\_\_ *x*-intercepts when graphed. | 3 |
| P 4 | *Use the image to answer the question.*  A curve with 3 inflection points is plotted on a coordinate grid. The plot starts in quadrant 2, descends into quadrant 3, rises to a turning point on the x-axis left of the origin, descends to a turning point in quadrant 4 and rises through quadrant 1.  Find the zeros of the polynomial function , given its graph. Enter the zeros in order from least to greatest. Enter any non-integer numbers in decimal form.  The zeros of j(x) are x = \_\_\_, x=\_\_\_, and x=\_\_\_ | -2; -0.5; 0.5 |
| P 5 | *Use the image to answer the question.*  A curve with 2 turning points is plotted on a grid and is labeled q left parenthesis x right parenthesis. The plot starts in quadrant 3, rises to a turning point in quadrant 2, drops to a turning point in quadrant 3, and rises through quadrants 4 and 1.  Find the missing values in the factored form of q(x), given its graph. | 2; 1 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Factor to solve the equation . |  |
| Q 2 | What are the additional linear factors of if is a factor? |  |
| Q 3 | Which of the following best describes how you can determine the *x*-intercepts of a polynomial function without graphing? | Find the zeros of the polynomial function. |
| Q 4 | *Use the image to answer the question.*  A curve with 2 turning points is plotted on a coordinate grid and is labeled g left parenthesis x right parenthesis. The plot starts in quadrant 3, rises to a turning point in quadrant 2, drops to a turning point in quadrant 4 and rises through quadrant 1.  Find the zeros of , given its graph. |  |
| Q 5 | *Use the image to answer the question.*  A curve with 2 turning points is plotted on a coordinate grid and is labeled m left parenthesis x right parenthesis. The plot starts in quadrant 3, rises to a turning point in quadrant 1, drops to a turning point in quadrant 4 and rises through quadrant 1.  Which of the following could be the equation of in factored form, given its graph? |  |

**Lesson 3 – Repeating Roots**

**Key Words:**

* **Factor Theorem** – the theorem that states if c is a zero of a polynomial, then is a factor of that polynomial
* **linear factor** – a factor in the form ax±c where a and c are constants; the exponent on the variable is 1
* **multiplicity** – the number of times a given factor appears in the factored form of the equation of a polynomial
* **root** – the value of x that makes an algebraic expression equal to zero
* **zeros of the polynomial** – the input (x) values that make a polynomial have a value of zero

**Formulas:**

* Factor Theorem: If is a zero, then is a factor of the polynomial

**Objective 1:** In this section, you will identify the multiplicities of linear factors of polynomial equations.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas**:

* Recall that you can find where a polynomial touches the *x*-axis by finding the *zeros of the polynomial*, or *roots*, of the *linear factors* of a polynomial.
* Different polynomials can have the same zeros, but the graph is different.
* The polynomials in the following example have the same roots at , but each graph has a different shape.
  + Notice that the factor appears once in and twice in -raised to the power of 2.

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* The *multiplicities* of each linear factor affect the shape of the graph.
  + The value of the exponent on the linear factor is the *multiplicity* of that factor.
  + Let’s look at how the *multiplicity* of a linear factor affects the graph.

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| **Graph Behavior** |
| If a factor is raised to an **odd** exponent, the graph will cross the *x*-axis. |
| If a factor is raised to an **even** exponent, the graph will touch the *x*-axis and turn back to the direction it came from. |

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| **Example:** Describe the zeros and describe the behavior of the polynomial at its *x*-intercepts. |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | | **Linear Factor** | **Zero/Root** | **x-intercept** | **Multiplicity** | **Behavior** | |  |  |  | 1 | Odd: cross the *x*-axis | |  |  |  | 3 | Odd: cross the *x*-axis | |  |  |  | 4 | Even: touch the *x*-axis and turn back | |

**Objective 2:** In this section, you will construct polynomial functions given the zeros and the multiplicity of each.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas:**

* You can construct a polynomial from the zeros and multiplicity.
* To do so, we will use the *Factor Theorem* to identify the factors and behavior of the graph to identify the multiplicities of those factors.

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| **The Factor Theorem**  If is a zero, or *root*, of a polynomial, then is a factor of that polynomial. |

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| **Example:** Construct a possible polynomial from the following graph. | |
| **Step 1:** Identify the zeros of the graph. These will be the *x*-values where the graph crosses or touches. | The graph has zeros at: |
| **Step 2:** Apply the Factor Theorem to the zeros to identify the factors of the polynomial. | |  |  |  | | --- | --- | --- | | **Zero ()** | **Factor ()** | **Factor Simplified** | |  |  |  | |  |  |  | |  |  |  | |  |  |  | |
| **Step 3:** Examine the behavior at each zero to determine if the multiplicity of the factor is even or odd.  [Recall](#Bookmark1) the rules for graph behavior. | |  |  |  |  | | --- | --- | --- | --- | | **Zero** | **Factor** | **Graph Behavior** | **Multiplicity** | |  |  | **Odd**: The graph crosses the *x*-axis at –4. | Assign an exponent of 1. | |  |  | **Even**: The graph touches the *x*-axis andturns back at –1. | Assign an exponent of 2. | |  |  | **Odd**: The graph crosses the *x*-axis at 3. | Assign an exponent of 1. | |  |  | **Even**: The graph touches the *x*-axis andturns back at 5. | Assign an exponent of 2. | |
| **Step 4:** Construct the polynomial using the factors and multiplicity of those factors. | is a possible polynomial expression.  Note that this is just one option of a polynomial that can be constructed. The multiplicities can change if they remain even or odd as assigned. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Identify the multiplicities of the linear factors of .  The multiplicity factor of (x-1) is \_\_\_. The multiplicity factor of (x+4) is \_\_\_. The multiplicity factor of (2x+5) is \_\_\_. | 2; 1; 1 |
| P 2 | *Use the image to answer the question.*  A plotted curve with 3 inflection points is labeled m left parenthesis x right parenthesis. The plot starts in quadrant 2, descends to the x-axis, rises to a turning point in quadrant 1, descends to a turning point in quadrant 4 and rises through quadrant 1.  Consider the graph of m(x). Given that (x-4) is a factor of m(x), is the multiplicity of (x-4) even or odd?  Option #1: even  Option #2: odd | 2 |
| P 3 | *Use the image to answer the question.*  A plotted curve with 3 inflection points is labeled m left parenthesis x right parenthesis. The plot starts in quadrant 2, descends to the x-axis, rises to a turning point in quadrant 1, descends to a turning point in quadrant 4 and rises through quadrant 1.  Consider the graph of m(x). Given that (x+1) is a factor of m(x), which of the following course be the multiplicity of (x+1)?  Option #1: -1  Option #2: 0  Option #3: 1  Option #4: 2 | 4 |
| P 4 | Which of the following could be the multiplicity of a zero whose graph crosses the *x*-axis?  Option #1: 2  Option #2: 3  Option #3: 4 | 2 |
| P 5 | Greg was asked to construct a polynomial function with the following zeros and multiplicities. Which function should he write?    Function #1:  Function #2:  Function #3: : | 2 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Identify the multiplicities of the linear factors of . | The multiplicity of (x + 4) is 1, the multiplicity of (x − 1) is 2, and the multiplicity of (x + 3) is 5. |
| Q 2 | *Use the image to answer the question.*  A plotted curve with 4 inflection points is labeled p left parenthesis x right parenthesis. The plot starts in quadrant 3, rises steeply to a turning point in quadrant 2, before dropping and rising twice in quadrant 1.  Consider the graph of . Given that is a factor of , is the multiplicity of even or odd? Why? | The multiplicity of is even because the graph of touches the *x*-axis at (3, 0) and then turns around. |
| Q 3 | *Use the image to answer the question.*  A plotted curve with 4 inflection points is labeled p left parenthesis x right parenthesis. The plot starts in quadrant 3, rises steeply to a turning point in quadrant 2, before dropping and rising twice in quadrant 1.  Consider the graph of the polynomial function . Given that is a factor of , which of the following could be the multiplicity of ? | 1 |
| Q 4 | Jaime was asked to construct a polynomial function with the following zeros and multiplicities. Which function should she write? |  |
| Q 5 | Which of the following could be the multiplicity of a zero whose graph touches the x-axis then turns back? | 2 |

**Lesson 4 – Factoring Techniques**

**Key Words:**

* **common factor** – a constant, variable, or polynomial that is a factor of every term in a polynomial
* **greatest common factor (GCF)** – the largest constant and/or the variable with the largest exponent that all terms have in common
* high-degree polynomial – a polynomial of degree 3 or higher
* **linear factor** – an expression in the form , that can be factored from the function f(x), where a and c are constants and the exponent on the variable x is 1
* **polynomial identity** – a polynomial equation that is always true for any value of the variables
* **quadratic** – a polynomial of the second degree
* **quadratic factor** – an expression of the form of which is a factor of a polynomial f(x)
* **zeros of a polynomial** – the input (x) values that make a polynomial have a value of zero

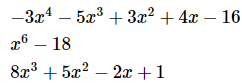
**Formulas:**

* Quadratic Formula:
* Polynomial Identities:

**Objective 1:** In this section, you will factor high-degree polynomials by decomposing them into quadratic factors.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas**:

* Recall that you previously used division to decompose polynomials with *linear factors*.
* We will now focus on *high-degree polynomials* that may not always be able to be decompose into *linear factors*.
  + A *high-degree polynomial* is a polynomial of 3 degrees or higher.
  + For example:
  + 
* You can factor a *high-degree polynomial* by decomposing the function into *quadratic factors* and then decomposing them further by factoring the quadratics.
  + Recall that a quadratic factor is expressed in the form .
  + Polynomial division can be used to decompose a *high-degree polynomial* into two factors if one factor is known.
* Once you have decomposed a *high-degree polynomial* into one or more *linear factors* and a *quadratic*, you can use one of these methods to find the remaining zeros of the *quadratic*:
  + factoring
  + completing the square
  + using the quadratic formula

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| **Example:** Given that is one factor, decompose into its factors and find the zeros of the polynomial. | |
| **Step 1:** Use polynomial division to decompose the polynomial into two factors. | Polynomial (dividend):  Known factor (divisor): |
| **Step 2:** Rewrite the polynomial as the two factors. |  |
| **Step 3:** Find the zero of the linear factors. | One zero of the polynomial is . |
| **Step 3:** Find the zeros of the quadratic factor using one of the methods – factoring, completing the square, or the quadratic formula. | Quadratic factor:  Using the quadratic formula:  where      The zeros of the quadratic factor are and . |
| **Step 4:** Put the zeros of the linear factor and quadratic factor together and state the answer. | The zeros of the original polynomial  are  . |

**Objective 2:** In this section, you will factor high-degree polynomials by grouping.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

* Recall that when you factor a polynomial, any *common factor* can be factored out to help reduce the polynomial. However, in high-degree polynomials, not all terms have a *common factor*.
* You can factor *high-degree polynomials* by factoring out a *greatest common factor (GCF)* of **groups of terms** in a polynomial instead of factoring a *GCF* from all terms.
  + *Greatest Common Factor (GCF)*: the largest constant and/or variable with the largest exponent that all terms have in common.
  + For example, the polynomial is . When is factored out, the statement becomes .

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| **Example:** Factor the high-degree polynomial by grouping and then identify the zeros. | |
| **Step 1:** Examine the polynomial for a common factor. | No common constant or variable can be divided out of all terms. |
| **Step 2:** Check for pairs of terms that can be grouped for factoring. | Notice that the first two terms (and ) have a common factor of .  The third and fourth terms (and ) have a common factor of . |
| **Step 3:** Group the terms and factor out the GCF of each group to rewrite the polynomial. |  |
| **Step 4:** Examine the new polynomial expression for a GCF. | The common factor is now . |
| **Step 5:** Factor out the new GCF to rewrite the polynomial expression. | The high-degree polynomial is now factored by grouping. |
| **Step 6:** Find the zeros of the polynomial by setting each factor to 0 and solving for *x*. | First Factor:  Second Factor: |
| **Step 7**: State the solution. | The solutions, or zeros, to the high-degree polynomial are |

**Objective 3:** In this section, you will factor high-degree polynomials by using their structures to identify patterns.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

* Recall the following polynomial identities:
  + When polynomials factor to just linear factors
    - Perfect square binomial:
    - Perfect square binomial:
    - Difference of two squares:
  + When polynomials factor a a linear factor and a quadratic factor:
    - Difference of perfect cubes:
    - Sum of perfect cubes:
* Often, a pattern of a high-degree polynomial may not immediately be obvious in the structure but factoring out a GCF (greatest common factor) may reveal the pattern you need.

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| **Example 1:** Identify the pattern in the structure of the high-degree polynomial and factor the polynomial. | |
| **Step 1:** Decompose the polynomial by finding the GCF and rewriting the expression. | Both terms have a common factor of . If you factor out from both terms, the expression becomes: |
| **Step 2:** Examine the factor inside of the parentheses to determine if it can be rewritten to fit a polynomial identity pattern. | can be written as which fits the pattern .  Rewritten expression: |
| **Step 3:** Use the polynomial identity to factor the expression. | Identity:  let and |
| **Step 4:** State the answer. | The factored form of the original high-degree polynomial is: |

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| **Example 2:** Factor the high-degree polynomial equation by finding the pattern in its structure and solving the equation. | |
| **Step 1:** Identify the pattern and polynomial identity. | The polynomial appears to have the structure of **.** |
| **Step 2**: Rewrite the original equation to fit the polynomial identity pattern. |  |
| **Step 3:** Identify the values of a and b and substitute to find the factors. | Let and  The factored form of is . |
| **Step 4:** Use the factored form to find the zeros. |  |
| **Step 5:** State the answer. | The real solution is **.** |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Which of the following is a factor of , given that *x* is one of the linear factors?  Option #1:  Option #2:  Option #3: | 3 |
| P 2 | Your friend Chadwick is trying to factor the polynomial . "I noticed that I could group the first and the third terms; this would help me begin to factor the polynomial,” says Chadwick. You notice that another equally valid grouping would be to group the first and second terms, with a common factor of , and the third and fourth terms, with a common factor of 5. Given that both of these first steps are valid, which of the following options is **not** equivalent to Chadwick’s polynomial?  Option #1:  Option #2:  Option #3: | 3 |
| P 3 | Factor the expression by grouping.    Then, fill in the missing pieces in the factorization of . | 9; 2 |
| P 4 | Identify the pattern that can be used to factor the high-degree polynomial .  Pattern #1:  Pattern #2:  Pattern #3: | 2 |
| P 5 | Factor the high-degree polynomial expression ￼ and then solve for the value of *x*.  *x* = \_\_\_ | -3 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Renee is asked to find a factor of , given that *x* is one of the linear factors. Which of the following is another correct factor? |  |
| Q 2 | Which of the following is the GCF of the polynomial ? |  |
| Q 3 | Which of the following is a complete set of solutions to ? | 0, 3, -6 |
| Q 4 | Which pattern can be used to factor the high-degree polynomial ? |  |
| Q 5 | Factor the high-degree polynomial to solve the polynomial equation . |  |

**Lesson 5 – Zeros of Polynomials**

**Key Words:**

* **degree** – the value of the highest exponent on any variable in a polynomial
* **leading coefficient** – the coefficient (the numerical portion) of the first term of a polynomial
* **linear factor** – a factor in the form ax±c where a and c are constants; the exponent on the variable is 1
* **multiplicity** –the number of times a given factor appears in the factored form of the equation of a polynomial
* **zeros of a polynomial** – the input (x) values that make a polynomial have a value of zero

**Objective 1:** In this section, you will describe the relationship between the degree of a polynomial and the number of zeros it has.

*Mathematical Practice Standard: Construct viable arguments and critique the reasoning of others.*

**Big Ideas**:

* When a polynomial factors as a product of *linear factors*, the *multiplicities* of the factors add up to the degree of the polynomial.
* The number of *zeros for a polynomial* is less than or equal to the *degree* of the polynomial.
* Examine the following polynomials and graphs.
  + Notice that all of them are third-degree polynomials.
  + Notice that each function has a different number of *x*-intercepts, where the graph crosses or touches the *x-*axis.

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| **one factor**  **one zero/root**  **one *x*-intercept** | **two factors**  **two zeros/roots**  **two *x*-intercepts** | **three factors**  **three zeros/roots**  **three *x*-intercepts** |
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| **State the degree and the number of zeros for each polynomial.** | |
| Example 1:     |  |  |  | | --- | --- | --- | | **Factor** | **Zero/Root** | **Multiplicity** | |  |  | 1 | |  |  | 1 | |  |  | 2 |  * The multiplicities add to three, therefore the degree of the polynomial is three. * The polynomial has three zeros | Example 2:     |  |  |  | | --- | --- | --- | | **Factor** | **Zero/Root** | **Multiplicity** | |  |  | 3 | |  |  | 2 | |  |  | 1 |  * The multiplicities add to 6, therefore the degree of the polynomial is six. * The polynomial has three zeros |

**Objective 2:** In this section, you will use the zeros of a polynomial function to sketch its graph.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas:**

* A polynomial equation can be analyzed to determine how the graph must be drawn.
* Recall the following things that you have learned about polynomials;
  + decomposing polynomials to find linear factors
  + using linear factors to find zeros of a polynomial
  + using the multiplicity of the factors to determine the behavior of the function
* Recall that the *multiplicity* of a factor determines the behavior of the polynomial at the factor’s root.
  + If a factor is raised to an **odd** exponent, the graph will cross the *x*-axis.
  + If a factor is raised to an **even** exponent, the graph will touch the *x*-axis and turn back to the direction it came from.

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| **Example:** Sketch the polynomial using the zeros of the polynomial. | |
| **Step 1:** Rewrite the standard form polynomial into factored form. | Notice that all terms have a GCF of –3 that can be factored out: |
| **Step 2:** Factor the trinomial inside of the parentheses. |  |
| **Step 3:** Rewrite the full polynomial. |  |
| **Step 4:** Determine the zeros and multiplicity of each factor. Use this information to determine other features of the graph such as the *x*-intercepts and behavior at the *x*-intercepts. | |  |  |  |  |  | | --- | --- | --- | --- | --- | | **Factor** | **Zero/Root** | **Multiplicity** | **Behavior** | **x-intercept coordinates** | |  |  | 1 | crosses the x-axis |  | |  |  | 1 | crosses the x-axis |  | |  |  | 1 | crosses the x-axis |  | |
| **Step 5:** Use the information to sketch a graph. |  |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Based on the degree, what is the greatest number of zeros the polynomial could have? | 4 |
| P 2 | Based on the degree of the polynomial , what is the greatest number of zeros it could have? | 4 |
| P 3 | Which of the following describes the appearance of the graph at a zero whose multiplicity is one? Enter the number of the correct option.  Option #1: The graph crosses the *x*-axis.  Option #2: The graph touches the *x*-axis and turns around.  Option #3: There is not enough information provided. | 1 |
| P 4 | Which of the following describes the appearance of the graph at a zero whose multiplicity is four? Enter the number of the correct option.  Option #1: The graph crosses the *x*-axis.  Option #2: The graph touches the *x*-axis and turns around.  Option #3: There is not enough information provided. | 2 |
| P 5 | How many times with the graph of cross the *x*-axis? | 2 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Which of the following properly describes the relationship between the degree of a polynomial and the number of zeros it has? | The degree of the polynomial is the greatest number of zeros the polynomial could have. |
| Q 2 | How many zeros does the function have? | 3 |
| Q 3 | Use the zeros to determine which of the following graphs is a sketch of the polynomial . | A curve passes through quadrants 2 and 4 on a coordinate plane with the x-axis ranging from negative 6 to 2 in unit increments. |
| Q 4 | Which of the following graphs is a possible sketch of the polynomial ? | A curve passes through quadrants 3 and 1 on a coordinate plane with the x-axis ranging from negative 5 to 5 in unit increments. |
| Q 5 | Which graph is a possible sketch of the polynomial ? | A curve passes through quadrants 3, 2, and 1 on a coordinate plane with the x-axis ranging from negative 5 to 5 in unit increments. |

**Lesson 6 – Relative Extrema of Polynomials**

**Key Words:**

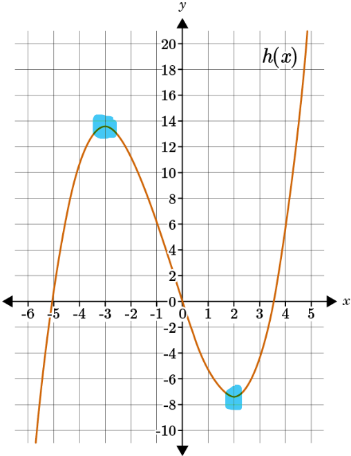
* **absolute maximum** – the largest value that a mathematical function can have over its entire domain
* **degree** – the value of the largest exponent on a variable in a polynomial
* **extrema** – plural of extremum
* **extremum** – a maximum or a minimum of a mathematical function
* **maxima** – plural of maximum
* **minima** – plural of minimum
* **relative (local) maximum** – a peak in the sketch of a polynomial in which the value at that peak is the highest value in that area
* **relative (local) minimum** – a valley in the sketch of a polynomial in which the value at the valley is the lowest value in that area
* **relative extrema** – the highest or lowest values on a given interval
* **turning point** – a point on the graph of a polynomial function where the changes from increasing to decreasing or vice versa
* **zeros of a polynomial** – the input (x) values that make a polynomial have a value of zero

**Objective 1:** In this section, you will identify relative maximum and minimum values on a polynomial graph.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas**:

* Recall the graph of a polynomial and the points on the graph when the polynomial turns or creates peaks and valleys. For example, the parts indicated in blue here:



* Notice in the graph above that the ends of the graph continue beyond what’s represented in the image.
* When it is not possible to determine the *absolute maximum* or *minimum* from the graph use the *relative (local) maximum and minimum*.
  + *absolute maximum* – the largest value that a function can have over its entire curve
  + *absolute minimum* - the smallest value that a function can have over its entire curve
  + *relative (local) maximum* – a peak/high point in one area of the graph
  + *relative (local) minimum* – a valley/low point in on area of the graph
  + If a graph has more than one peak or valley, use the plural *maxima* or *minima*.
  + The maximum and minimum values together are known as *extremum* or *extrema* for plural.
* Use the following example to find where the *minima* and *maxima* of this function occur and the value of each from the graph.

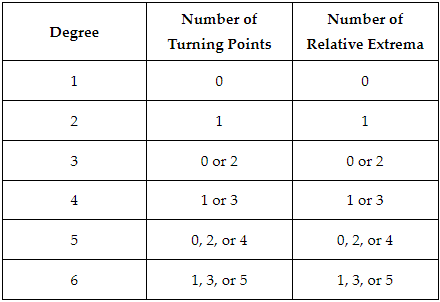
|  |  |  |
| --- | --- | --- |
| **Example** | **Maxima** | **Minima** |
|  | The graph shows two valleys or low points at and .  The graph continues up indefinitely, so these must be the lowest values on the graph. In this case, use the *absolute minima*.  Absolute Minima:  or  or | The graph shows one peak or high point at .  This is not the largest value the function can have because both sides rise toward infinity. In this case, use the *relative maximum*.  Relative Maximum:  or |

**Objective 2:** In this section, you will describe the relationship between the degree of a polynomial and the number of relative minima and maxima it has.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas:**

* Recall that the number of zeros for a polynomial is less than or equal to the degree of the polynomial.
* You can also relate the degree of a polynomial to the number of *relative extrema* that the graph of a polynomial will have.
  + For any polynomial, the greatest number of *relative extrema* will be one less than the degree of the polynomial.
  + The number of possible *relative extrema* decreases by two from the maximum possible number.



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| **Example:** Describe the relationship between the degree and the possible number of relative extrema of the polynomial . | |
| **Step 1:** Find the degree of the polynomial by adding the multiplicity of each factor. | |  |  | | --- | --- | | **Factor** | **Multiplicity** | |  | 1 | |  | 2 | |  | 5 |   The degree of the polynomial is 8. |
| **Step 2:** Determine the possible number of relative extrema. | Because the degree is 8, the function will have relative extrema.  It may have 1,3,5, or 7 relative extrema. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | *Use the image to answer the question.*  A horizontal S-shaped curve is plotted on a coordinate plane with the x-axis ranging from negative 4 to 4 in increments of 0.5 and the y-axis ranging from negative 3 to 4 in increments of 0.5.  What is the *x*-value of the relative minimum on the graph of ?  *x* = \_\_\_ | 1 |
| P 2 | *Use the image to answer the question.*  A horizontal S-shaped curve is plotted on a coordinate plane with the x-axis ranging from negative 5 to 5 and the y-axis ranging from negative 13 to 5 in unit increments.  What is the *x*-value of the relative minimum on the graph of ?  *x* = \_\_\_ | -3 |
| P 3 | *Use the image to answer the question.*  An M-shaped curve is plotted on a coordinate plane with the x-axis ranging from negative 4 to 4 in increments of 0.5 and the y-axis ranging from negative 4 to 4 in increments of 0.5.  What is the *y*-value of the absolute maximum on the graph of h?  *y* = \_\_\_ | 1 |
| P 4 | How many relative extrema can the polynomial have? | 2 |
| P 5 | What is the smallest degree a polynomial with five turning points could have? | 6 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | *Use the image to answer the question.*  A horizontal S-shaped curve is plotted on a coordinate plane with both axes ranging from negative 5 to 5 in unit increments.  Which of the following most accurately identifies the relative maximum and minimum of the polynomial function ? | There is a relative maximum at and a relative minimum at . |
| Q 2 | *Use the image to answer the question.*  A W-shaped curve is plotted on a coordinate plane with the x-axis ranging from negative 2 to 4 in increments of 0.5 and the y-axis ranging from negative 3 to 4 in increments of 0.5.  Carmen was asked to identify the relative extrema of the polynomial function . Which of the following statements should she make? | There is an absolute minimum at . |
| Q 3 | *Use the image to answer the question.*  A horizontal S-shaped curve is plotted on a coordinate plane with the x-axis ranging from negative 2 to 4 in increments of 0.5 and the y-axis ranging from negative 4 to 4 in increments of 0.5.  Brandon was asked to identify the relative maximum of the polynomial function . Which answer choice identifies the correct value(s)? | There is a relative maximum at . |
| Q 4 | Which of the following properly describes the relationship between the degree of a polynomial and the number of relative extrema it has? | The number of relative extrema of a polynomial is, at most, one less than the degree. |
| Q 5 | Rogelio is asked to sketch a graph of . How many turning points should his sketch include? | eight at most |

**Lesson 8 – Remainders**

**Key Words:**

* **Factor Theorem** – the theorem that states if c is a zero of a polynomial, then is a factor of that polynomial
* **linear factor** – a factor in the form ax±c where a and c are constants; the exponent on the variable is 1
* **remainder** – the final undivided part after division that is less or of lower degree than the divisor
* **Remainder Theorem** – a theorem stating that when a polynomial f(x) is divided by the linear polynomial, , the remainder of that division will be equivalent to f(a)

**Formulas:**

* Division Algorithm:
* Remainder Theorem: The value of a function,, at a given value, , then divide the polynomial by the linear function .
* Factor Theorem: If is a zero of a polynomial, then is a factor of that polynomial.

**Objective 1:** In this section, you will divide polynomials and represent the remainders.

*Mathematical Practice Standard: Attend to precision.*

**Big Ideas**:

* Previously, you used long division and synthetic division to reduce a polynomial to *linear factors*.
* The division algorithm helps express the result of a polynomial with a *remainder*.
  + Remainders can be integers or other polynomials.

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| **The Division Algorithm**   * The remainder is represented by adding it to the product of the quotient and divisor. * The degrees of the divisor and quotient will add up to the degree of the original polynomial.     where = polynomial, = divisor, = quotient, and = remainder |

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| **Example:** Divide the polynomial by , and express the result as the product of two polynomials and a remainder. | |
| **Step 1:** Match the first term of the divisor match the first term of the polynomial.  Put that term on top of the division bar above the term with the same degree. | To match the first terms you must multiply by . |
| **Step 2:** Multiply the divisor by **.**  Write the new expression below the first two terms of the polynomial, and subtract, placing parentheses around the new expression.  Bring down the next term of the dividend. | = |
| **Step 3:** Match the first term of the divisor with the first term of the new polynomial.  Put that term on top of the division bar above the term with the same degree. | To match the first terms you must multiply by . |
| **Step 4:** Multiply the divisor by .  Write the new expression below the second and third terms of the polynomial, and subtract, placing parentheses around the new expression.  Bring down the next term of the dividend. | = |
| **Step 5:** Match the first term of the divisor with the first term of the new polynomial.  Put that term on top of the division bar above the term with the same degree. | To match the first terms you must multiply by 8. |
| **Step 6:** Multiply the divisor by 8.  Write the new expression below the second and third terms of the polynomial, and subtract, placing parentheses around the new expression.  There is nothing in the dividend to bring down. | = |
| **Step 7:** Identify the divisor, quotient, and remainder. | The remainer, , is –1 because it can’t be brought to the top of the dividend.  The quotient, , is .  The divisor, , is |
| **Step 8:** Use the Division Algorithm to write the result of the division. |  |

**Objective 2:** In this section, you will apply the Remainder Theorem to evaluate polynomials.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* The *Remainder Theorem* is used to evaluate polynomials at a given input value.
  + The theorem makes it easier to evaluate polynomials because it only requires simple multiplication and addition.

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| **The Remainder Theorem**  When polynomial is divided by the linear polynomial , the remainder is the value of . In other words, the remainder is the value of the function at .  *How do I apply this in practice?*  If you want to find the value of a function,, at a given value, , then divide the polynomial by the linear function . |

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| **Example:** Use the Remainder Theorem to find the value of if . | |
| **Step 1:** Use the Remainder Theorem to identify what to divide the polynomial by. | * You are asked to evaluate at * This means that . * The divisor is . |
| **Step 2:** Perform polynomial division using the calculated divisor. |  |
| **Step 3:** State the answer. | Since the remainder is 10, the value of is 10. |

**Objective 3:** In this section, you will apply the Factor Theorem to find factors of polynomial functions.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* The *Remainder Theorem* gives the value of a polynomial when that polynomial is divided by a constant.
* [The *Factor Theorem*](#Bookmark6) says if the polynomial is divided by a constant , and the remainder is zero, then is a zero of the polynomial and is a factor of the polynomial.
  + The *Factor Theorem* can be used to quickly find factors and verify whether a value is a zero of a polynomial.
* If you divide a polynomial by a number and the *remainder* is zero:
  + The number is a zero of the polynomial.
  + It is also a factor of the polynomial.

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| **Example 1:** Is 4 a zero of ? | |
| **Step 1:** Determine what factor to divide the polynomial by. | If 4 is a zero of a polynomial, that means a factor could be . |
| **Step 2:** Perform division to identify the remainder. |  |
| **Step 3:** State the answer. | Notice the remainder is 234, which is not zero. Thus 4 is not a zero of . |

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| **Example:** Use the Factor Theorem to factor into a product of linear factors, given that 1 is a zero of . | |
| **Step 1:** Determine what factor to divide the polynomial by. | If 1 is a zero of a polynomial, that means a factor is . |
| **Step 2:** Perform division to identify the remainder. | The remainder is zero, proving that 1 is a zero of the polynomial. |
| **Step 3:** Write the new expression. |  |
| **Step 4:** Factor the second term. |  |
| **Step 5:** Put all factors together. |  |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | *Use the image to answer the question.*  x cubed plus 5 x squared plus 4 x plus 18 is divided by x plus 5 using the long division method.  Consider the polynomial division when . What is the value of ? | -2 |
| P 2 | *Use the image to answer the question.*  x squared minus 4 is divided by x minus 1 using the long division method. The first expression of divisibility, left parenthesis x squared minus x right parenthesis, is subtracted from the dividend. The remainder is x minus 4. The quotient is x.  The image shows the first few steps of the polynomial division when . Complete the polynomial division process, then give the value of *n(x)* when *x=1*. | -3 |
| P 3 | by to find .  *f(-1) = \_\_\_* | -2 |
| P 4 | Divide by to find . | 170 |
| P 5 | Use the Factor Theorem to factor into a product of linear factors, given that -3 is a zero of |  |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Divide the polynomial by . Then express the polynomial as the product of the divisor and the quotient plus the remainder. |  |
| Q 2 | Use the image to answer the question.  A long division has for its divisor 3 x superscript 4 baseline plus 9 x squared minus 13. The dividend is x squared minus 5 x.  The image shows the first few steps of the polynomial division of. Complete the division process and find the remainder of this polynomial division. | The remainder is . |
| Q 3 | Given , according to the Remainder Theorem, can be found by finding the remainder of which of the following division quotients? |  |
| Q 4 | Given the polynomial in its factored form as shown here: , apply the Factor Theorem. Which of the following is a zero of ? |  |
| Q 5 | Given a polynomial and that , apply the Remainder Theorem and then determine which of the following is a factor of . |  |

**Lesson 9 – Modeling with Polynomials**

**Key Words:**

* **interpret** – to give the answer in terms of the context of the equation
* **quantity** – any number or variable and any algebraic combination of the two (i.e. In , the quantities are x, 4, 8, and )
* **simple projectile motion** – the motion of an object that is subject only to gravity, neglecting air resistance, and has no means of propulsion on its own

**Formulas:**

* Single Motion Projectile:

**Objective 1:** In this section, you will represent polynomial relationships between two quantities.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* Polynomials are used to model many real-world situations and make predictions about that situation.
* You can use a polynomial to model situations that relate two quantities.
  + For example, a polynomial can relate the value of differing amounts of money (quantity one) deposited over time (quantity two).

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| **Example:** Linn is saving for a car. She wants to invest money each year for **three years** in an account that pays interest at the end of each year.   * At the start of the first year, she deposits $650. * At the start of the second year, she deposits $800. * At the start of the third year, she deposits $550.   Use where the second quantity, *r,* is the interest rate paid each year.  Write a model polynomial, , to find the value of Linn’s account at the end of three years. | |
| **Step 1:** Begin to set up the model by creating a term for each year that Linn has deposited money. | * The money deposited in the first year, $650, will earn interest three times. * The money deposited in the second year, $800, will earn interest for two time. * The money deposited in the third year, $550, will earn interest once. |
| **Step 2:** Add these up to create the model. |  |

* You can use a quadratic polynomial to represent the distance an object travels over time.
  + *Projectile motion* is an example of this and is modeled by a quadratic polynomial that relates the first quantity, height, and the second quantity, time.
  + This relationship is modeled by the following formula:

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| **Simple Projectile Motion**   * is the gravitational constant where the object is traveling.   + For Earth: or * initial velocity of the object * initial height of the object from the ground |

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| **Example:** A football is kicked from a tee on the ground with an initial velocity of 100 feet per second.  Write a model, , that represents two quantities:   1. the height of the ball from the ground, in feet, for 2. the time in terms of seconds after the ball is kicked   Use the formula for projectile motion: | |
| **Step 1:** Identify the variable values to be used in the projectile motion formula. | * Because the ball is on earth, and the problem is asking for measurement in feet per second, the gravitational constant is: * The initial velocity is given: * The ball is kicked from the ground, so the initial height is: |
| **Step 2:** Input the values into the formula to represent the model. |  |
| **Step 3:** Simplify the model. | This model represents the relationship between height, , and time, . |

**Objective 2:** In this section, you will interpret polynomial relationships between two quantities.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* To *interpret* the polynomial relationship between two *quantities*, you will perform two actions.
  + 1. Evaluate equations that represent the relationship between those quantities.
  + 2. Interpret the result in context of the relationship between quantities.
* Recall the [previous example](#Bookmark7) of Linn’s model for saving up for a car. Let’s evaluate the model and interpret the meaning of the relationship between the quantities.

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| **Example:** In the [previous example](#Bookmark7) we found a model for Linn to save up for a car. The model is . Suppose Linn’s savings pays 2.5% at the end of each year. Interpret the polynomial relationship that gives the value of the investment after three years at an interest rate of 2.5% paid at the end of each year.  Recall that which represents 1 plus the interest rate . | |
| **Step 1:** Substitute the value of the interest into the equation .  Reminder: The interest rate is expressed as a %. You will need to convert *r* to a decimal. |  |
| **Step 2:** Substitute the value of x into the model, . |  |
| **Step 3:** Simplify. |  |
| **Step 4:** Interpret the result. | With an interest rate of 2.5%, after three years the total amount of the savings will be $2,104.23. |

* Recall the [previous example](#Bookmark8)about kicking a football:

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| **Example:** A football is kicked from a tee on the ground with an initial velocity of 100 feet per second. After **three seconds**, how high off the ground is the football?  We found the model for this situation to be: | |
| **Step 1:** Review the model. | This model represents the relationship between height, , and time, . |
| **Step 2:** Substitute the value of into the model. |  |
| **Step 3:** Simplify. |  |
| **Step 4:** Interpret the result. | After 3 seconds, the football will be 156 feet above ground. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | The area of a rectangle whose width is 3 less than half its length can be found by the polynomial . Which of the following are the two related quantities represented by this polynomial? Enter the number of the correct quantity.  Quantity #1: length of the rectangle and width of the rectangle  Quantity #2: width of the rectangle and perimeter of the rectangle  Quantity #3: perimeter of the rectangle and area of the rectangle  Quantity #4: area of the rectangle and length of the rectangle | 1 |
| P 2 | A ball is dropped from a 30-foot-tall building, meaning it has no initial velocity. Write a model that represents the height of the ball from the ground, in feet, *t* seconds after it is dropped from the building. | 1. -16 2. 0 3. 30 |
| P 3 | The equation represents the relationship between the area and the base of a right triangle whose height is 8 inches. Write a sentence comparing the area and the triangle’s base when and .  When the base of the right triangle measures \_\_\_\_\_ inches, its area is \_\_\_\_\_ square inches. | 5; 20 |
| P 4 | Anders throws a baseball straight up from a height of 3 feet above the ground. The initial velocity of the baseball is 25 feet per second. What is the height of the ball 1.5 seconds after it is thrown? Round your answer to the nearest hundredth.  \_\_\_ seconds | 4.5 |
| P 5 | For the past 3 years, Kaira has deposited $600 at the beginning of each year into an investment account with an interest rate of 4.75%. Use , where *r* is the interest rate, and the equation . The equation represents the relationship between , the value of the investment after 3 years. Given that the amount of interest earned is the difference between the total value of the investment after 3 years and the sum of her $600 deposits, find the amount of interest that Kaira will earn at the end of the 3-year term. Round the answer to two decimal places.  The amount of interest that Kaira will earn at the end of the 3-year term is $\_\_\_\_\_. | 176.48 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | A ball is thrown directly upward from the ground with an initial velocity of 4.8 ft./sec. Represent the height of the ball from the ground *t* seconds after it was thrown upward using the model ). |  |
| Q 2 | Use , where *r* is the interest rate paid each year. Write a model polynomial, . Represent the final amount of a 7-year investment if $5,000 was deposited at the beginning of the first year, and $2,000 was deposited at the beginning of the third year. |  |
| Q 3 | The equation represents the relationship between the area in square units and the width of a rectangle whose length is 5 units longer than its width. Select the sentence that describes an accurate relationship between *A* and *w*. | increases as *w* increases when . |
| Q 4 | The equation represents the relationship between the volume of a sphere and its radius. What does the end behavior tell you about the relationship between the volume of the sphere and its radius? | As the radius increases to infinity, the volume of the sphere will increase to infinity. |
| Q 5 | Brayton wants to invest his high school earnings for the next 7 years. He deposits $1,500 into an investment account at the beginning of the first year, $2,000 at the beginning of the second year, $1,870 at the beginning of the third year, and $2,230 at the beginning of the fourth year. Use , where *r* is the interest rate, and the equation . The equation represents the relationship between , the value of the investment after 7 years, and its annual interest rate, *r*. Find the value of the investment account if the interest rate is 2.85%. | $8,840.80 |