Algebra 2

**Periodic Functions**

**Unit Summary:** In this unit you will graph equations with trigonometric functions. You will explore the relationship between evaluating the sine and cosine functions on the unit circle and graphing those same functions on the coordinate plane. You will also learn about the key characteristics of these function graphs and how to change them.

**GeoGebra:** [Amplitude, period and phase of trigonometric function – GeoGebra](https://www.geogebra.org/m/QsyyePBs)

**Lesson 2 – Periodicity**

**Key Words:**

* **cycle** – a sequence of events that is repeated at regular intervals
* **periodic** – a description of a cyclical phenomenon, i.e., one that has a repeating pattern
* **periodicity** – a characteristic of a cyclical phenomenon, i.e., one that has a repeating pattern

**Objective 1:** In this section, you willinvestigate periodic phenomena and interpret the descriptions.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* A situation is considered *periodic* when there is a sequence of repeating events. In other words, a *cycle* that occurs repeatedly. These scenarios are described as having *periodicity*.
* When given a description of a *periodic* phenomenon it should be analyzed in the following way:
  + Identify the unique events that occur in one *cycle*.
  + Identify the maximum and minimum values, a middle value, and how and when they occur.
  + Describe the *cycle* as a sequence of events, showing how the last event is the same as the first.

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| **Example 1:** At the North Pole, the sun rises once around March 20 and stays up for 6 months. Then it sets around September 22 and stays down for 6 months. One full year-long cycle consists of 6 months of daylight and 6 months of darkness. What can be said in terms of the percentage of sunlight at the North Pole? |
| * The maximum percentage of sunlight is 100% and the minimum percentage is 0%. Both the maximum and the minimum occur once per cycle. * Numerically, 50% would represent half of the percentage of sunlight, but in this analysis, either it is daylight, or it is dark. * The cycle is 100% daylight for 6 months, then 0% daylight for 6 months |

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| **Example 2:** One tidal cycle includes the rising of water to reach high tide and the receding of water to reach low tide. Since two cycles occur every 24 hours, 50 minutes, 1 cycle takes 12 hours, 25 minutes to complete. What else can be said about the rising and falling of tides in terms of water level? |
| * At high tide, the water is at its highest height, and at low tide it is at its lowest height. * Both heights occur once every cycle. * As the water recedes after high tide, it will reach a middle height. As it rises after low tide, it will also reach the middle height.   To illustrate the cycle:   * At low tide, the water begins rising. * At 3 hours, 6.25 minutes, it reaches its middle height. * At 6 hours, 12.50 minutes, it reaches high tide. * Then it starts to recede. * At 9 hours, 18.75 minutes it again reaches its middle height. * Then finally, at 12 hours, 25 minutes, low tide is reached once again |

**Objective 2:** In this section, you will identify graphs of periodic phenomena.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* *Periodic* phenomena can be represented with graphs in a variety of different shapes.
* *Periodic* graphs can be quickly identified because a distinct pattern occurs repeatedly.
  + To visualize one *cycle*, choose a point and trace the graph until you reach the next corresponding point.
  + For example, if you choose the maximum point, one full cycle is when you reach the next maximum point.

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| **Examples of Periodic Graphs** | |
| **Graph** | **Cycle** |
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| **Example:** The moon orbits Earth regularly. Depending on the day, the moon may be fully visible, partially visible, or barely visible. The amount of visibility is determined by the lunar cycle. Use the graph to describe the lunar cycle. | |
| How does the graph illustrate the length of the lunar cycle? | The pattern repeats at 30 days, so the lunar cycle is 30 days long. |
| During which days of the lunar cycle is the moon half visible? | The moon is 100% visible on day 0 and 0% visible on day 15, so it must be 50% visible on day 7.5 and also on day 22.5. |
| During which days of the lunar cycle is the moon one-quarter visible? | The moon is one-quarter visible on days 11.25 and 18.75. |
| During which days of the lunar cycle is the moon three-quarters visible? | The moon is three-quarters visible on days 3.75 and 26.25. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Which description depicts a periodic scenario? | the water level as the tide flows in and out |
| P 2 | Suppose high tides for a location occur at 3:30 a.m. and 4:00 p.m. When would a low tide likely occur? | 9:45 a.m |
| P 3 | Use the image to answer the question.  Does this graph appear to model a periodic situation? | Maybe. Without seeing more than one cycle, it is unclear what happens out to the left and out to the right. |
| P 4 | *Use the image to answer the question.*  How many complete cycles are shown in the graph?  There are \_\_\_\_ complete cycles shown in the graph. | 6 |
| P 5 | *Use the image to answer the question.*  What does the 𝑥-axis represent in this scenario? | the height of the yo-yo user's hand |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Which description depicts a periodic scenario? | the distance of a metronome from center as it keeps time |
| Q 2 | In an experiment, the temperature of a liquid is measured as it is heated and then allowed to cool. The temperature rises to 176°F and drops to 68°F at each step of the process. This process is repeated several times. What type of pattern does this temperature change follow? | The temperature follows a repeating pattern. |
| Q 3 | The height of a rider above the ground on a Ferris wheel follows a periodic pattern. What is a reasonable length of a cycle? | 10 minutes |
| Q 4 | Use the image to answer the question.  If this graph is periodic, what would you expect to see for 𝑥-values less than −2 and greater than 20? | The exact same pattern would repeat itself from both the left and the right. |
| Q 5 | Use the image to answer the question.  Is this a possible graph of periodic phenomena? | Yes, because the graph appears to have a cycle that repeats at regular intervals. |

**Lesson 3 – The Sine & Cosine Function Graphs**

**Key Words:**

* **amplitude** – one-half of the difference between the minimum and maximum values of the graph
* **continuous function** – a function whose graph does not have any breaks, gaps, or jumps
* **cycle** – one complete repetition of a pattern
* **frequency** – the number of periods of a periodic function in one unit on the x-axis; calculated as 1 period
* **midline** – in trigonometry, the horizontal line midway between the maximum and minimum values of a periodic function; the line about which the graph of a periodic function oscillates
* **period** – the horizontal length of each cycle of a periodic function
* **periodic** – a characteristic of a graph that means it has a repeating pattern
* **x-intercept** – the point where the graph crosses the x-axis
* **y-intercept** – the point where the graph crosses the y-axis

**Formulas:**

* Amplitude:
* Frequency:

**Objective 1:** In this section, you will use a table of values to graph sine and cosine functions.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* Recall the unit circle which will be helpful when graphing sine and cosine function.
  + Recall that in the unit circle, **cosine refers to the *x-*coordinate** and **sine refers to the *y*-coordinate**.

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| **Unit Circle** |
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* Graphing trigonometric functions involves using the unit circle and a table of values.
  + The starting input value will always be 0.
  + The ending value is found by setting the theta value equal to and solving for theta.
    - Example 1: , you will set .
    - Example 2: , you will set .
    - Example 3: , you will set .
  + The output values are found by substituting each input value into the original equation. This is the point in the process where you will use the [unit circle](#Bookmark1).
* Follow the steps for graphing trigonometric functions.

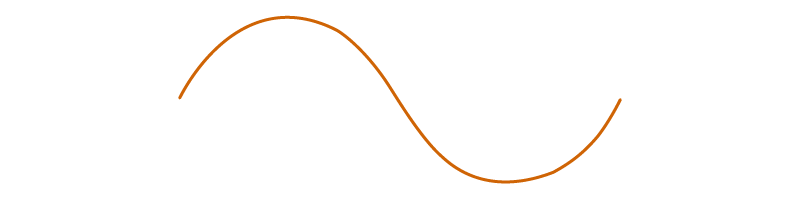
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| **Example:** Using a table of values, graph the following trigonometric function . Label the amplitude, period, and midline. | |
| **Step 1:** The first input of the table will be 0, but we need to find the ending value by setting the and solving for theta. | Theta is solved for and equals . This will be the final value in your table from 0 to .   |  |  | | --- | --- | | **Finding the input values** | (input) | | Start value | 0 | |  |  | |  |  | |  |  | | End value |  | |
| **Step 2:** Determine the halfway point on the table. | |  |  | | --- | --- | | **Finding the input values** | **(input)** | | Start value | 0 | |  |  | | Determine the halfway point between 0 and |  | |  |  | | End value |  | |
| **Step 3:** Determine the remaining input values of the table. | |  |  | | --- | --- | | **Finding the input values** | **(input)** | | Start value | 0 | | Determine the halfway point between 0 and |  | | Determine the halfway point between 0 and |  | | Determine the halfway point between and |  | | End value |  | |
| **Step 4:** Substitute each theta value into the original function to find the *y*-values. | Use the unit circle to evaluate sine at different points. Recall that sine refers to the y-coordinate when looking at the unit circle.   |  |  |  | | --- | --- | --- | |  |  |  | | 0 |  | 0 | |  | \*On the unit circle, . | 2 | |  | \*On the unit circle, . | 0 | |  | \*On the unit circle, . | -2 | |  |  | 0 | |
| **Step 5:** Create an input-output table with the input and output values that you have found. | |  |  | | --- | --- | |  |  | | 0 | 0 | |  | 2 | |  | 0 | |  | -2 | |  | 0 | |
| **Step 6:** Graph the table of values on a coordinate grid. | Since the inputs jump in value by adding , this can be the scale on the *x-*axis. |
| **Step 7:** Identify the amplitude, midline, and period. | * The amplitude of the graph is 2 * The midline is * The period is |

**Objective 2:** In this section, you will identify the amplitude, period, and frequency of the sine and cosine functions using their graphs.

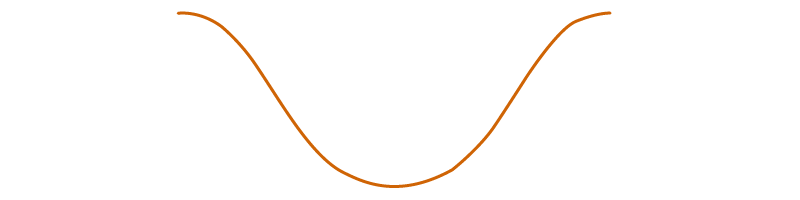
*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* You can use the graphs of sine and cosine functions to find their *amplitude*, *period*, and *frequency*.
* The *amplitude* is determined by finding one-half of the difference between the maximum and minimum values of the graph.
* The *period* is found by tracing the graph at , then moving to the right until one cycle is completed. You can use the following shapes of the sine and cosine functions as a guide when tracing the graph.
  + The *period* can also be found by finding the distance between two consecutive maximum and minimum values.
  + Sine Function



* + Cosine Function



* The *frequency* can be identified by dividing 1 by the period: frequency=

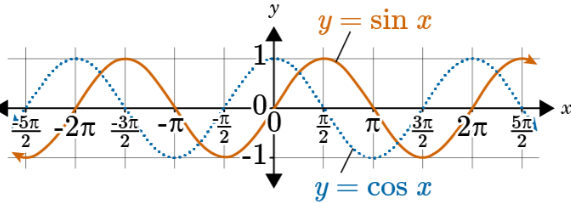
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| **Example:** Find the amplitude, period, and frequency of the graph of the function. | |
| **Step 1:** Find the amplitude.   * Locate the max and min values of the graph:   + max value is 3   + min value is –3 * Calculate the amplitude using the formula.     The amplitude of the graph is 3. | |
| **Step 2:** Find the period.   * Begin at and trace the graph using the shape for one cycle of the graph of a sine function. * The period is the distance between the origin and the last point of the cycle, as highlighted in the following image.      * The last point of the cycle is . It’s distance from the origin is . Thus, the graph has a period of . | |
| **Step 3:** Find the frequency.   * Use the formula for frequency: * The frequency of the function is . | |

**Objective 3:** In this section, you will compare the attributes of the sine and cosine functions based on their graphs.

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**Big Ideas:**

* Sine and cosine functions have many attributes that can be analyzed by looking at their graphs.
* The differences between the two functions occur at their intercepts, since their graphs cross the *x-* and *y*-axes at different points.
* The following table outlines the features of the basic sine and cosine functions.



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| **Feature** | **Sine Function** | **Cosine Function** |
| Period |  |  |
| Amplitude | 1 | 1 |
| Frequency |  |  |
| Domain | set of all real numbers | set of all real numbers |
| Range |  |  |
| Maximum Value | 1 | 1 |
| Minimum Value | -1 | -1 |
| x-intercepts | or where *n* is an integer | or where *n* is an odd integer |
| y-intercepts |  |  |

* The attributes of both and can change if they undergo a transformation.
  + The periods will take on different values if the graphs of the basic sine and cosine functions become narrower or wider or if they are moved horizontally.
  + The amplitude, range, and max/min values will be affected if the graphs are shifted vertically or if the graphs become taller or shorter.
  + Only the domain of the functions remains the same even if a transformation is applied.

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| **Example:** Find the attributes of the following function. | |
| Amplitude |  |
| Period | * The graph does not cross the origin and is therefore a cosine function. * Trace the graph starting at (0,3) moving right. * One cycle of the graph is at point (. * The distance between (0,3) and along the x-axis is . * Thus, the period is . |
| Frequency | Frequency = |
| Domain/Range | * Domain: set of all real numbers * Range: The graph is bounded from and . The range is . |
| Max/Min | Maximum: 3  Minimum: 3 |
| x-intercepts | * The graph crosses the *x-*axis at or where *n* is an odd integer. * Since the graph continues infinitely in both the left and right directions, it will cross the *x*-axis multiple times. |
| y-intercepts | The graph crosses the *y*-axis once at . |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | To graph sine functions, create a table of values. Fill in the missing values for the table that would be used to graph . | 3; -3 |
| P 2 | Use the image to answer the question.  Identify the amplitude of the sine function. | 2 |
| P 3 | *Use the image to answer the question.*  Which of the following elements has a value of 4 in the graph of the function? Enter the option number of the correct answer.  Option #1: frequency  Option #2: period  Option #3: amplitude  Option #\_\_\_ has a value of 4. | 3 |
| P 4 | *Use the image to answer the question.*  What is the range of the graphed function?  \_\_\_\_ ≤ *y* ≤ \_\_\_\_ | -3; 1 |
| P 5 | Use the image to answer the question.  Which of the following statements about this graph is correct?  Statement #1: The ranges of the functions are equal.  Statement #2: The domain of the sine function is smaller than that of the cosine function.  Statement #3: The periods of the functions are equal. | 3 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Use the table to answer the question.    Use the table of values to graph the  and find the amplitude, midline, and period. | amplitude = 4, midline: *y* = 0, period = |
| Q 2 | Use the image to answer the question.  Based on the set of points in the graph, determine the equation that would match the graph. |  |
| Q 3 | Use the image to answer the question.  Identify the amplitude, period, and frequency of the graphed function. Which of these statements about the function is correct? | The graph has a period of 𝜋. |
| Q 4 | Use the image to answer the question.  Identify the amplitude, period, and frequency of the graphed function. Which of these statements about the function is correct? | The graph has a frequency of . |
| Q 5 | Use the image to answer the question.  Compare the attributes of the graphed sine and cosine functions. Based on your comparisons, which of the following statements is correct? | The sine function has a smaller y-intercept than the cosine function. |

**Lesson 4 – Reflections**

**Key Words:**

* **even function** – a function that satisfies the equality f(−x) =f(x) for all values of x; a function whose graph is symmetric about the y-axis
* **odd function** – a function that satisfies the equality f(−x) =−f(x) for all values of x; a function whose graph is symmetric about the origin
* **parent function** – the simplest function in a family of functions
* **reflection** – a transformation that flips or mirrors a figure or a graph over a line

**Formulas:**

* [Unit Circle](#Bookmark12)

**Objective 1:** In this section, you will graph periodic functions of the form or

using tables of values.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* To apply a *reflection* to the graph of one function, you can use a table of values to create ordered pairs that correspond to the reflected image.
* The negative sign in the function or indicates that its graph is a *reflection* of the graph of or across the *x-*axis.
* When a *reflection* across the *x*-axis is applied, the *x*-coordinate stays the same and the sign of the *y*-coordinate changes.

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| **Example:** Reflect the function to obtain the graph of the function . | |
| **Step 1:** Create a table of values for the function . To find the *y*-value, evaluate the function at each value of *x*. Create the ordered pair . |  |
| **Step 2:** Plot the points on a coordinate plane to obtain the graph of the basic sine function . |  |
| **Step 3:** Create a table of values for by changing the sign of the *y-*coordinate. Create a new set of ordered pairs with the new *y*-coordinates. |  |
| **Step 4:** Plot the new ordered pairs to graph . |  |
| Observe the two graphs on the same coordinate plane. | |

**Objective 2:** In this section, you will graph periodic functions of the form or

using tables of values.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* The negative sign in the function or indicates that its graph is a *reflection* of the graph of or across the *y-*axis.
* The sine function is an *odd function* because it satisfies the equality .
  + And their graphs are the same.

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| Evaluate the function at . |  |

* The cosine function is an even function because it satisfies the equality .
  + And their graphs are the same.

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| Evaluate the function at . |  |

**Objective 3:** In this section, you will describe periodic functions in terms of the reflections specified in their equations.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* [Recall](#Bookmark2) the attributes of basic sine and cosine functions.
* These attributes can change once a transformation is applied to the graphs of the basic sine and cosine functions.
  + All transformed sine functions belong to the same family of sine functions.
  + All transformed cosine functions belong to the same family of cosine functions.
* Since all transformed functions are derived from the basic periodic functions, and , are called *parent functions*.
* A transformation changes the graph of the *parent function* and adds parameters that can help you identify what transformation was applied to the *parent function* to obtain the transformed function.
* Recall that a transformation flips a figure or a graph over a line called the line of reflection, either the *x-* or *y-*axis.
  + *x-*axis: The graph is flipped vertically, and the point becomes the point .
    - In general, the function becomes .
    - The negative sign is placed in front of the function.
  + *y-*axis: The graph is flipped horizontally, and the point becomes the point .
    - In general, the function becomes .
    - The negative sign is placed in front of the input, *x*.

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| **Example:** Describe the transformation that has been applied to obtain the periodic function . | |
| **Step 1:** Identify the parent function. | The parent function of is . |
| **Step 2:** Analyze where the negative sign is in the transformed function. | The negative sign in the function comes before the input x, which indicates that the function has been reflected over the *y-*axis. |
| **Step 3:** Observe the graph of both functions. | When you graph and flip its graph horizontally, you will end up with the graph of . |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | The table contains values for 𝑓(𝑥). If ℎ(𝑥) is the reflection of 𝑓(𝑥) across the x-axis, find the missing value in the ordered pair below, for ℎ(𝑥).    ( | -1 |
| P 2 | Evaluate the function  at . | 1 |
| P 3 | Which of the following statements accurately describes a difference between the graphs of f𝑓(𝑥)=sin(𝑥) and 𝑔(𝑥)=sin(−𝑥)?  Statement #1: If (𝑥,𝑦) is a point in 𝑓(𝑥), then (𝑥,−𝑦) is a point in 𝑔(𝑥).  Statement #2: The function g(𝑥) is the same as f(𝑥).  Statement #3: If (𝑥,𝑦) is a point in f(𝑥), then (−𝑥,−𝑦) is a point in 𝑔(𝑥).  Statement #\_\_\_\_ is correct. | 1 |
| P 4 | Which of the following accurately describes the transformation from  and ?  Statement #1: The function 𝑔(𝑥) is a reflection of 𝑓(𝑥) across the y-axis.  Statement #2: The function 𝑔(𝑥) is a reflection of 𝑓(𝑥) across the x-axis.  Statement #3: The function 𝑔(𝑥) is a reflection of 𝑓(𝑥) across 𝑦=𝑥.  Statement #\_\_\_\_ is correct. | 1 |
| P 5 | What is ? |  |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Using , find the y-values in the second row of the table by substituting each x-value into the function. Which of the following is the graph of ? |  |
| Q 2 | Using f, find the y-values in the second row of the table by substituting each x-value into the function. Which of the following is the graph of ? |  |
| Q 3 | Using , find the y-values in the second row of the table by substituting each x-value into the function. Which of the following is the graph of ? |  |
| Q 4 | Which of the following could be found in a table of values for the function ? |  |
| Q 5 | When the function  is reflected across the x-axis, the resulting function is . If the coordinates of the y-intercept of are (0,1), what are the coordinates of the y-intercept of the reflected function ? | (0, -1) |

**Lesson 5 – Changes in Amplitude**

**Key Words:**

* **amplitude** – one-half of the difference between the maximum and minimum values of the graph
* **key points** – the five important points that make up one cycle of the graph of sine and cosine functions; these include the maximum point(s), minimum point(s), and x-intercepts

**Formulas:**

* Amplitude =

**Objective 1:** In this section, you will graph periodic functions of the form or using a table of values.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* To graph any sine and cosine function you need the five *key points* that make up one cycle of the function.
  + Since the functions are periodic, you can use the pattern or shape made by these *five key points* to draw two or more cycles of the graph.

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| **Five Key Points** | |
| Sine Function  For the graph of , the five key points are:   * three *x-*intercepts * one minimum point * one maximum point |  |
| Cosine Function  For the graph of the function , the five key points are:   * two x-intercepts * one minimum point * two maximum points |  |

* The *amplitude* of the graphs of and changes when a vertical stretch or compression is applied to its graph.
  + A vertical stretch makes the graph taller, increasing the *amplitude*.
  + A vertical compression makes the graph shorter, decreasing the *amplitude*.

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| **For the function and , the following applies:** |
| * + If , its graph is obtained by vertically stretching the graph of or , which makes its amplitude greater than the parent function.   + If , its graph is obtained by vertically compressing the graph of or , which makes the amplitude less than that of the parent function. |

* Since vertical stretch or compression makes the graph of the parent function taller or shorter, the *y-*values of the parent function are affected.
  + Each *y-*coordinate of the ordered pairs of the parent function is to be multiplied by the factor *a*.
  + The good thing about the parent functions and is that the y-coordinates of the five key points of their graphs are either –1, 0, or 1, and are easily multiplied by the factor *a*.
* You can create a table of values to graph the function in the form and .
  + Keep the x-coordinate of the parent function
  + multiply each y-coordinate by the factor a
  + create ordered pairs and plot them on the coordinate plane
  + connect them using a smooth curve
  + extend the graph in the left and right directions

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| **Example:** Graph the function . | |
| **Step 1:** Analyze the function and identify the value of *a*. | The function takes the form , with .  Since the value of a is greater than 1, the graph of is obtained by vertically stretching the graph of its parent function . |
| **Step 2:** Use the [five key points](#Bookmark3) of the graph to create a table of values for . | Multiply each y-value by 2, then create the ordered pair .   |  |  |  |  | | --- | --- | --- | --- | |  |  | \*multiply each y-coordinate by 2. | Ordered Pairs | | 0 | 0 |  |  | |  | 1 |  |  | |  | 0 |  |  | |  | -1 |  |  | |  | 0 |  |  | |
| **Step 3:** Plot each ordered pair and connect the plotted points using a smooth curve. | Since the graph of is obtained by vertically stretching the graph of its parent function by a factor of 2, its amplitude is greater than that of . |

**Objective 2:** In this section, you will describe the effects on periodic functions when a < > 1 in equations of the form or .

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* If you encounter a function in the form or the value of *a* affects the amplitude, maximum and minimum values, and range of the function as follows:

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| **Value of *a*** | **Example** |
| * amplitude: * min value: * max value: * range: |  |
| * amplitude: * min value: * max value: * range: |  |
| * amplitude: * min value: * max value: * range: |  |

**Objective 3:** In this section, you will write equations of periodic functions with various amplitudes on the basis of a graph, a table of values, or a verbal description.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* Recall what you have learned about transformations of sine and cosine functions.
  + [By attaching a coefficient other than 1](#Bookmark4) to the equation of a sine or cosine function, you can make its graph taller or shorter.
  + [Recall](#Bookmark5) that a transformation flips a figure or a graph over a line called the line of reflection, either the *x-* or *y-*axis.
* Using this knowledge, you can work backwards from a graph, table of values, or verbal description o f a periodic function to write the equation of the function.
* Recall that the *amplitude* of a function is half the distance between the max and min values.

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| **Example:** What is the amplitude of the function whose values are shown in the following table, and what is the equation of this function based on its amplitude? | |
| **Step 1:** Identify the maximum value and minimum value. | The maximum value is and the minimum value is . |
| **Step 2:** Apply the formula for amplitude. | Amplitude=  The amplitude is . |
| **Step 3:** Identify if the function is sine or cosine. | The function passes through the origin, so it is a sine function. |
| **Step 4:** Create the equation for the function. | The equation for the function described in the table is . |

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| **Example 2:** What is the amplitude of this function, and what is the equation of this function based on its amplitude? | |
| **Step 1:** Identify the maximum value and minimum value. | The maximum value is 3 and the minimum value is –3. |
| **Step 2:** Apply the formula for amplitude. | Amplitude = |
| **Step 3:** Identify if the function is sine or cosine. | The function does not pass through the origin and appears to be a cosine function. |
| **Step 4:** Identify other potential transformations. | Instead of starting at a maximum of and falling to a minimum of , it starts at a minimum of and rises to a maximum of .  The shape has been flipped across the x-axis, so the equation of the function requires a negative sign in front. |
| **Step 5:** Create the equation for the function. | * Reflected over x-axis   The equation for this function is . |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Complete the table of values that can be used to graph . |  |
| P 2 | Use the image to answer the question.  Consider the graph and determine its type of function.  Enter 1 if the function is of the form .  Enter 2 if the function is of the form .  Find the amplitude a of the function.  Function type: \_\_\_\_  Function amplitude: *a* = \_\_\_\_\_ | 1; 3 |
| P 3 | Which of the following functions would have the largest maximum?  Option #1:  Option #2:  Option #3: | 3 |
| P 4 | Use the image to answer the question.  What is the amplitude of function shown in the graph? Enter the correct option number.  Option #1: 1.5  Option #2: −1.5  Option #3: 3  Option #4: −3  The option with the correct amplitude is Option # \_\_\_\_. | 1 |
| P 5 | Use the table to answer the question.    Given the values in the table, determine the amplitude of the cosine function.  The amplitude of the cosine function is \_\_\_\_\_. | 4 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Use the image to answer the question.  Create a table of values to determine which is the correct graph of . | Graph B |
| Q 2 | Which of the following is the graph of ? |  |
| Q 3 | Consider the functions and how does the amplitude of *g(x)* compare to the amplitude of *f(x)*? | The amplitude of *g(x)* is two times the amplitude of *f(x)*. |
| Q 4 | Use the image to answer the question.  Write the equation of the periodic function based on the graph. |  |
| Q 5 | Use the table to answer the question.    Write the equation of the periodic function based on the table. |  |

**Lesson 6 – Changes in Period & Frequency**

**Key Words:**

* **amplitude** – one-half the difference of the maximum value and the minimum value of the graph
* **frequency** – the number of periods of a periodic function in one unit on the x-axis; calculated as 1 period
* **horizontal compression** – a transformation that causes the graph of a function to shrink toward the y-axis when all the x-coordinates are multiplied by a factor of a, where a>1
* **horizontal stretch** – a transformation that causes the graph of a function to stretch away from the y-axis when all the x-coordinates are multiplied by a factor a, where 0<a<1
* **maximum** – the largest value of the function
* **minimum** – the smallest value of the function
* **parent function** – the simplest function in a family of functions
* **period** – the horizontal length of each cycle of a periodic function
* **periodic function** – a function whose graph has a repeating pattern
* **transformation** – a change in the size, shape, position, or orientation of a graph
* **x-intercept** – the point where the graph crosses the x-axis

**Formulas:**

* Period:
* Frequency:
* Calculate *b* given the period:

**Objective 1:** In this section, you will use tables of values to graph the sine or cosine of *bx*, that is, *x* with a coefficient different from 1.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* Adding a coefficient to *x* in the functions or causes *horizontal compression* or *stretching* in the graph of the transformed function.
  + A sine or cosine function with a coefficient greater than 1 produces a narrower or more *compressed* graph.
    - The *period* is reduced while the *frequency* increases.
  + A sine or cosine function with a coefficient less than 1 produces a wider or more *stretched* graph.
    - The *period* is increased while the *frequency* is reduced.
* The period of a sine or cosine function can be found using the equation of the function itself.
  + If a function has the form or :
  + The period is
  + The frequency is

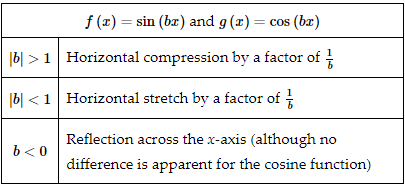
|  |  |
| --- | --- |
| **Function** | **Graph** |
| * The cycle of a basic cosine function, or parent function, is completed over an interval of . * Period: * Frequency: |  |
| * The function has been compressed horizontally so that a single cycle is completed over an interval of rather than over an interval of . * The period is the period of the parent function.   + Period: * Notice that there are now 4 cycles over the interval of . * The frequency is 4 times greater than it was in the parent function.   + Frequence: |  |
| * The function has been stretched horizontally so that a single cycle is completed over an interval of . * The period is 3 times the period of the parent function.   + Period: * Notice that there is now of a cycle over the interval of . * The frequency is what it was in the parent function.   + Frequency: |  |

**Objective 2:** In this section, you will describe the effects on periodic functions when *b* is not equal to 1 in equations of the form and .

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* Graphs of functions or are transformations of their *parent function* graphs: each is a *horizontal stretch or compression* by a factor of .



* The *period* of sine and cosine functions is determined by the following formula:
  + period =

|  |  |
| --- | --- |
| **Example:** Describe the graph of as a transformation of its parent function, determine the period of , then graph and the parent function . | |
| **Step 1:** Determine the value of *b* from the transformed function and identify the transformation. | and   * The value of b in the transformed function is greater than 1 and it is also negative. * The transformation is the composition of a reflection across the x-axis and a horizontal compression by a factor of . |
| **Step 2:** Determine the period. |  |
| **Step 3:** Graph the transformed function and the parent function. |  |

**Objective 3:** In this section, you will write equations of periodic functions with periods not equal to 2 given their graphs, tables of values, or verbal descriptions.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* [Recall](#Bookmark6) that when a *horizontal stretch or compression* by a factor of *b* is applied, graphs of functions and have period of .
* Given the equation of a sine or cosine function, you can determine its *period* using this formula.
* Knowing the factor of a *horizontal stretch or compression* you can determine its period and write the equation of a function without having to graph it.
* You can calculate *b* if you know the *period* by using the formula:

|  |  |
| --- | --- |
| **Example 1:** Write the equation of the cosine function whose graph is shown below. | |
| **Step 1:** Find the period. | From the graph, you can see that one cycle is between 0 and .  You can also see from the graph that this is a cosine function.  period = |
| **Step 2:** Calculate *b*. |  |
| **Step 3:** Write the equation of the function. | * We know it is a cosine function. So, the parent function is . * *b* is calculated to be . |

* You can also write the equation of the periodic function given its table of values.

|  |  |
| --- | --- |
| **Example 2:** Write the equation of the periodic function represented int he table. | |
| **Step 1:** Determine the function. | Since , the table represents a sine function. |
| **Step 2:** Find the period. | Use two consecutive zeros and **double** the horizontal distance. Recall that the *x* values represent horizonal distance.  Zeros:  Period = |
| **Step 3:** Find *b*. |  |
| **Step 4:** Write the equation. | * We know that it is a sine function. * We know the value of *b* is . |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Use the image to answer the question.  What is the value of b in the function  in the graph?  *b* = \_\_\_\_\_ | 4 |
| P 2 | Use the image to answer the question.    What is the value of b in the function  in the graph?  *b* = \_\_\_\_\_ |  |
| P 3 | If 𝑏>1, will the period of a function in the form  be less than or greater than 2π?  Option #1: If b>1, then the period will always be less than 2π.  Option #2: If b>1, then the period will always be greater than 2π. | 1 |
| P 4 | Use the image to answer the question.  Write the equation of the trigonometric function that is represented in the graph.  f(x)=cos (\_\_\_\_\_) |  |
| P 5 | Use the table to answer the question.    Write the equation of the periodic function represented in the table.  f(x)=sin(\_\_\_\_\_) |  |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Use the image to answer the question.  Identify the function in the graph. |  |
| Q 2 | Use the image to answer the question.  Identify the function in the graph. |  |
| Q 3 | Describe the graph of the function  as a transformation of its parent function. | The graph is a horizontal compression by a factor of . |
| Q 4 | Which is an equation of the cosine function that has a period of ? |  |
| Q 5 | Use the image to answer the question.  What is the equation of the trigonometric function in the graph? |  |

**Lesson 7 – Phase Shifts**

**Key Words:**

* **maximum** – the largest value of the function
* **minimum** – the smallest value of the function
* **parent function** – the simplest function in a family of functions
* **period** – the horizontal length of each cycle of a periodic function
* **periodic** – a characteristic of a graph that means it has a repeating pattern
* **periodic function** – a function whose graph has a repeating pattern
* **phase shift** – the measure of how far the graph of the parent sine and cosine functions are shifted horizontally
* **transformation** – a change in the size, shape, position, or orientation of a graph
* **x-intercept** – the point where the graph crosses the x-axis

**Objective 1:** In this section, you will graph periodic functions of the form using tables of values.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* [Recall](#Bookmark3) that you need five key points to graph one complete cycle of a sine or cosine function.
  + x-intercepts, and minimum and maximum points
* Once you graph one complete *cycle*, which corresponds to the *period* of a function, you can repeat the pattern to the left and right as many times as needed.
* Recall the *parent functions* with their corresponding key points:

|  |  |
| --- | --- |
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* The graph of a function in the form of or represents a horizontal translation, or *phase shift*, of a corresponding parent function.
  + If , the graph is a horizontal translation of ***c* units to the left.**
  + If , the graph is a horizontal translation of **c units to the right.**
* To graph *phase shifts*:
  + Horizontally translate five key points of the parent function by *c* units
  + Connect the points with a smooth curve
  + Repeat the pattern to extend the graph

|  |  |
| --- | --- |
| **Example:** Graph the function . | |
| **Step 1:** Identify the phase shift. | The function has a phase shift of , which corresponds to the horizontal translation by units to the right. |
| **Step 2:** Make a table of values for the key points by adding to the x-coordinates of the parent functions five key points. |  |
| **Step 3:** Plot the five key points of the function and connect them with a smooth curve. Repeat the pattern to extend the graph. |  |

**Objective 2:** In this section, you will describe the transformations of periodic functions in the form when c is greater or less than 1.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* [Recall](#Bookmark7) that by adding or subtracting some number *c* from *x* in a sine or cosine function, you can shift the function left or right. This is known as a *phase shift*.
  + Shift to the left *c* units: or
  + Shift to the right *c* units: or
* After a *phase shift* has been applied, the phases of the sine or cosine waves don’t change at the same points as they do in the *parent functions* and .
  + They don’t reach a maximum or minimum at the same places along the *x-*axis.

|  |
| --- |
| **Example:** What transformation of produces the following graph? How do you know? |
| The cosine function has been shifted by units to the right, so the equation is . This can be identified by looking at the graphs maximum point of which has been shifted from its parent functions maximum of . |

**Objective 3:** In this section, you will write equations of periodic functions containing phase shifts given graphs, tables of values, or verbal descriptions.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* Recall that a full cycle, or phase, is radians. Meaning, the graph of a cosine or sine function repeats itself every units.

|  |  |
| --- | --- |
| Observe the following phase shift of which has been shifted to the right.   * The graph has been shifted to the right by . * The origin is now at instead of . * The shifted graph can be represented by the equation . |  |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Use the image to answer the question.  What transformation of the parent sine function is shown on the graph?  shift by \_\_\_\_\_ |  |
| P 2 | Complete the table so that all five key points for the graph of  can be shown in the graph. | 1. , 1 2. 0 3. 0 |
| P 3 | The graph of sin (*x*+*c*) is shifted  units to the right compared to sin 𝑥. What is the value of c? |  |
| P 4 | The graph of sin (*x*+*c*) is shifted  units to the left compared to sin 𝑥. What is the value of c? |  |
| P 5 | Which of the following sine functions is equivalent to ?  Option #1:  Option #2:  Option #3: | 2 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | What is the phase shift of the function . |  |
| Q 2 | Use the image to answer the question.  Which of the following functions correctly represents the graph of the sine function given in the preceding image? |  |
| Q 3 | Describe how the graph of the function  differs from the graph of sin *x*. | It shifts the graph of sin *x* to the right by units. |
| Q 4 | Use the table to answer the question.    The values of a sine function at intervals of  have been recorded in the table.  Identify the value of the phase shift (𝑐) and write the equation of the function. |  |
| Q 5 | Use the image to answer the question.  Determine the equation of the sine function shown in the graph. |  |

**Lesson 8 – Changes in Midline**

**Key Words:**

* **cosine** – a trigonometric function that, for an acute angle, is the ratio between the leg adjacent to the angle when it is considered part of a right triangle and the hypotenuse
* **midline** – in trigonometry, the horizontal line midway between the maximum and minimum values of a periodic function; the line about which the graph of the periodic function oscillates
* **sine** – the trigonometric function that, for an acute angle, is the ratio between the leg opposite the angle when it is considered part of a right triangle and the hypotenuse

**Objective 1:** In this section, you will use a given table of values to graph a trigonometric function in the form and identify the midline.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* The *midline* of a sine or cosine function is the horizontal line halfway between the maximum and the minimum *y-*values.
  + Recall that the *midline* for the parent functions, and , is the *x-*axis.
  + Recall that, to calculate the *midline*, you take the average of the maximum and minimum *y-*values.



* A *vertical phase shift* will move the function up or down, which results in a change to the *midline*.
* Functions in the form or have a midline at , where can be positive or negative.
* Creating a table of values can help us to visualize the *midline* of a sine or cosine function that has a *vertical shift.*

|  |  |
| --- | --- |
| **Example:** Create a table of the five key points of the function . Graph the function and identify the maximum, minimum, and the midline. | |
| **Step 1:** Complete the table of the five key points by plugging in the x-values into the equation to find the corresponding values of . |  |
| **Step 2:** Use the five key points to graph the function. | Key points: |
| **Step 3:** Identify the maximum and minimum. | maximum:  minimum: |
| **Step 4:** Calculate the midline by taking the average of the *y*-values from the maximum and minimum. | The midline of the transformed function is . |

**Objective 2:** In this section, you will describe the effects on periodic functions when d is not equal to 0 in equations of the form .

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* [Recall](#Bookmark8) the midline of a sine or cosine function in the form or is . The value of can be positive or negative.
* The graph of the periodic functions in these forms is obtained by translating the graph of or up when , or down when .

|  |  |
| --- | --- |
| **Observe the graphs of a family of sine functions with different values for .** | |
|  | |  |  | | --- | --- | | **Sine Function** | **Midline** | |  |  | |  |  | |  |  | |

|  |  |
| --- | --- |
| **Observe the graphs of a family of sine functions with different values for .** | |
|  | |  |  | | --- | --- | | **Cosine Function** | **Midline** | |  |  | |  |  | |  |  | |

**Objective 3:** In this section, you will write equations of periodic functions with midlines not on the x-axis given their graphs, table of values, or verbal descriptions.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* [Recall](#Bookmark9) that the *midline* of the parent sine and cosine function is , or the x-axis. The *midline* changes when the graph is *shifted vertically*.
* You can use the *midline* to write the equation of a sine or cosine function given their graph or table of values.
  + Given a graph: find the *midline* by drawing a horizontal line halfway between the maximum and minimum points of the graph.
  + Given a table of values: find the maximum and minimum points from the table and average the *y*-values to get the midline.

|  |  |
| --- | --- |
| **Example 1:** Write the equation of the periodic function represented by the graph. | |
| **Step 1:** Identify the type of function. | The graph looks to be a **sine** function that has been shifted down. |
| **Step 2:** Draw a horizontal line halfway between the maximum and minimum points of the graph. |  |
| **Step 3:** Identify the midline. | The midline of the graph is . |
| **Step 4:** Write the equation of the function. | * We know that the function is a sine function. * We know the midline is at –3. |

|  |  |
| --- | --- |
| **Example 2:** Write the equation of the periodic function represented by the table of values. | |
| **Step 1:** Identify the type of function. | * One of the maximum values, according to the table, is at . * Since the y-value at is a maximum, this table of values represents a cosine function. |
| **Step 2:** Find the midline by averaging the *y-*values of the maximum and minimum. | * maximum: * minimum: * midline:   The midline of the transformed function is . |
| **Step 3:** Write the equation of the function. | * We know that it is a cosine function. * We know the midline is . |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Use the table to answer the question.    Review the table of values to determine the midline for the sine function.  The equation of the midline is *y* = \_\_\_\_\_. | -3 |
| P 2 | Fill in the equation for a sine curve with a midline shift up 6 units. | 6 |
| P 3 | Use the image to answer the question.  What is the value of d in the function  in the function shown in the graph?  The value of d in the function  is \_\_\_\_\_. | 2 |
| P 4 | A periodic function has a maximum value of 2.5 and a minimum value of −3.5. What is the midline of its graph?  *y* = \_\_\_\_\_. | -0.5 |
| P 5 | Complete the equation of the periodic function represented by the table of values by entering the midline. | 6 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Which of the following tables of values would you use to graph the function ? |  |
| Q 2 | Use the image to answer the question.  What is the midline of the cosine function? |  |
| Q 3 | How does the graph of  compare with the parent function ? | The graph is shifted 12 units down. |
| Q 4 | Use the image to answer the question.  Given the graph, write the equation of the periodic function. |  |
| Q 5 | Use the image to answer the question.  Write the equation for the trigonometric function based on its graph. |  |

**Lesson 9 – Simultaneous Transformations of Periodic Functions**

**Key Words:**

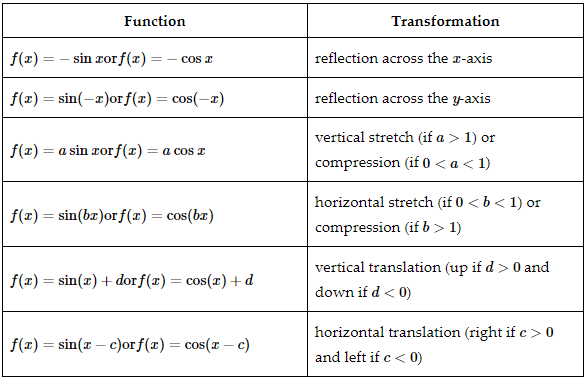
* **amplitude** – one-half of the distance between the maximum and minimum values of the graph
* **midline** – in trigonometry, the horizontal line midway between the maximum and minimum values of a periodic function; the line about which the graph of a periodic function oscillates
* **period** – the length of one cycle
* **phase shift** – the measure of how far the graph of the parent sine and cosine functions are shifted horizontally

**Objective 1:** In this section, you will create the graphs of periodic functions using transformations.

*Mathematical Practice Standard: Model with mathematics*

**Big Ideas**:

* Recall the different transformations that can be applied to the graphs of parent functions and .



* If you notice many of the above parameters have been added to a sine or cosine function, then you know that multiple transformations have been applied to the graph of the parent functions.
* Any sine and cosine functions can be written in the form and . Each parameter in the equation represents a specific transformation outlined below.

|  |  |
| --- | --- |
| **Any sine and cosine functions can be written in the form:**  **and .**  Each parameter in the equation represents a specific transformation outlined below. | |
| Vertical Stretch/Compression () | * : vertical stretch * : vertical compression * : reflection across the *x-*axis   Vertical stretch and compression affect the amplitude.  The amplitude of the graphs of periodic functions is . |
| Horizontal Stretch/Compression () | * : horizontal stretch * : horizontal compression * : reflection across the *y-*axis   Horizontal stretch and compression change the period.  The period of the graphs of periodic functions is . |
| Horizontal Phase Shift () | * : horizontal shift units to the **right** * : horizontal shift units to the **left** |
| Vertical Shift () | * : vertical shift units **up** * : vertical shift units **down**   Vertical shifts or translations affect the midline of the graph.  The midline of the graphs of periodic functions is . |

* Use the following steps to graph sine and cosine functions with multiple transformations.
  + Rewrite the equation in the form or .
  + Identify the following order of transformations:
    - 1. horizontal shift/phase shift
    - 2. horizontal stretch/compression and/or reflection across x-axis
    - 3. vertical stretch/compression and/or reflection across y-axis
    - 4. vertical shift
  + Use the five key points of the graph of the parent sine or cosine function and apply each transformation successively to these points. You can also start with the graph of the parent function and apply each transformation successively to the graph.
  + Plot the points and connect them with a smooth curve

|  |  |
| --- | --- |
| **Example:** Use transformations to graph . | |
| **Step 1:** Rewrite the equation in the appropriate form. | is already written in the form . |
| **Step 2:** Identify the transformations applied. | * : vertical stretch by a factor of 3 * : no horizontal stretch or compression * : no phase shift is applied * : vertical shift down 2 units, the midline will be |
| **Step 3:** Start with the parent function graph. |  |
| **Step 4:** Apply the first transformation. | Vertical stretch by a factor of 3. |
| **Step 5:** Apply the next transformation. | Translate the graph 2 units down. |
| **Step 6:** Extend the graph in either direction to add more cycles. | Extend the graph. |

**Objective 2:** In this section, you will create the equations of periodic functions given their graphs.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* [Recall](#Bookmark10) the specific transformations that each parameter represents in periodic function of the form and .
* From the graphs of periodic functions, you can determine the values of *a, b, c,* and *d* which can be used to write the equation from the graph.
  + Find *a* by finding the graph’s *amplitude*. If the graph is reflected across the *x-*axis, then *a* should be negative.
  + Find *b* by using the *period* of the graph: . If the graph is reflected across the *y-*axis, then *b* should be negative.
  + Find *d* by determining the *midline* of the graph, .

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Determine the direction of the horizontal shift for the function .  Statement #1: The function has a right horizontal shift of  units.  Statement #2: The function has a left horizontal shift of  units.  Statement #3: The function has a right horizontal shift of  units.  Statement #\_\_\_\_ describes the correct shift. | 2 |
| P 2 | Determine the direction of the horizontal shift for the function .  Statement #1: The function has a phase shift right of  units.  Statement #2: The function has a phase shift left of  units.  Statement #3: The function has a phase shift left of  units.  Statement #\_\_\_\_ describes the correct phase shift. | 2 |
| P 3 | Use the image to answer the question.  Determine the equation of the midline of the periodic function in the graph.  𝑦 = \_\_\_\_\_ | 2 |
| P 4 | Use the image to answer the question.  Determine the amplitude and the equation of the midline of the periodic function in the graph.  amplitude: \_\_\_; 𝑦= \_\_\_\_ | 2; 3 |
| P 5 | Use the image to answer the question.  Write the equation of the function graphed using the cosine.  𝑓(𝑥) = \_\_\_\_\_ cos (\_\_\_\_\_) + \_\_\_\_\_ | 3; |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Samira is given the periodic function . If she wants to create a graph of the function, which of the following should she choose? |  |
| Q 2 | Use the image to answer the question.  Determine which function matches the given graph. |  |
| Q 3 | Use the image to answer the question.  Which function is shown on the graph? |  |
| Q 4 | Which is the graph of the function |  |
| Q 5 | Use the image to answer the question.  What is the equation of the midline for the function graphed? | *y* = 2 |

**Lesson 10 – Periodic Function Flexibility**

**Key Words:**

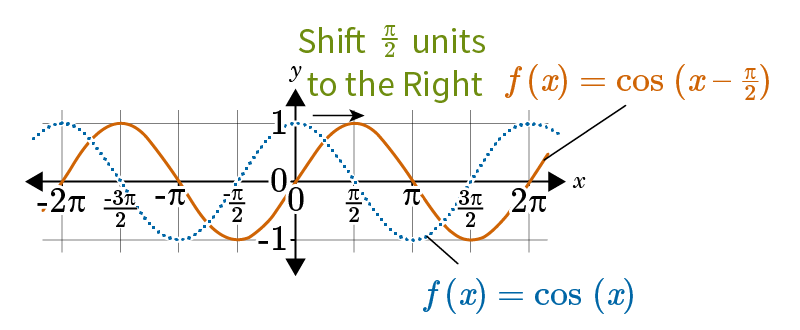
* **amplitude** – one-half of the distance between the maximum and minimum values of the graph
* **argument** – the independent variables that determine the value of a function
* **cosine** – a trigonometric function that, for an acute angle, is the ratio between the leg adjacent to the angle when it is considered part of a right triangle and the hypotenuse
* **midline** – in trigonometry, the horizontal line midway between the maximum and minimum values of a periodic function; the line about which the graph of a periodic function oscillates
* **parent function** – the simplest form of a family of functions
* **period** – the length of one cycle
* **periodic function** – a function that has a repeating pattern
* **phase shift** – the measure of how far the graph of the parent sine and cosine functions are shifted horizontally
* **relative (local) maximum** – a peak in the sketch of the graph of a function in which the value at that peak is the highest value in that area
* **relative (local) minimum** – a valley in the sketch of the graph of a function in which the value at the lowest point of that valley is the lowest value in that area
* **sine** – the trigonometric function that, for an acute angle, is the ratio between the leg opposite the angle when it is considered part of a right triangle and the hypotenuse
* **translation** – a transformation that shifts an image or set of points up, down, left, and/or right
* **vertical shift** – the measure of how far the graph of the parent sine and cosine functions are shifted vertically

**Objective 1:** In this section, you will rewrite the equations of sine functions as cosine functions.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* When the sine function is expressed as a cosine function, it becomes .
  + This is obtained by shifting the graph of by units to the right.
  + is one-fourth of the period of the sine function .



* In general, if you are given a sine function to be rewritten as a cosine function:
  + Find the period of the sine function
  + Translate the graph of the cosine function by units to the right, where the period is taken from the sine function
  + Apply other transformation if necessary

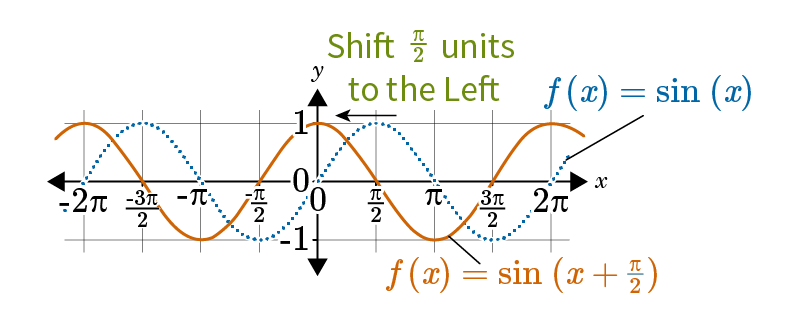
|  |  |
| --- | --- |
| **Example:** Express as a cosine function. | |
| **Step 1:** Find the period of the sine function. | Recall that the period can be found by .  Period = |
| **Step 2:** Divide the period found of the sine function by 4 to determine how many units the graph of is to be translated to the right. | Shifting the graph of by units to the right results in . |
| **Step 3:** Apply other transformations so that the two functions have the same graph. |  |
| **Step 4:** State the answer. | The graph of expressed as a cosine function is . These two functions will result in the exact same graph. |

**Objective 2:** In this section, you will rewrite cosine functions as translated sine functions.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* When the cosine function is expressed as a sine function, it becomes .
  + This is obtained by shifting the graph of by units to the left.
  + is one-fourth of the period of the cosine function .



* In general, if you are given a cosine function to be rewritten as a sine function:
  + Find the period of the cosine function
  + Translate the graph of the sine function by units to the left, where the period is taken from the cosine function
  + Apply other transformation if necessary

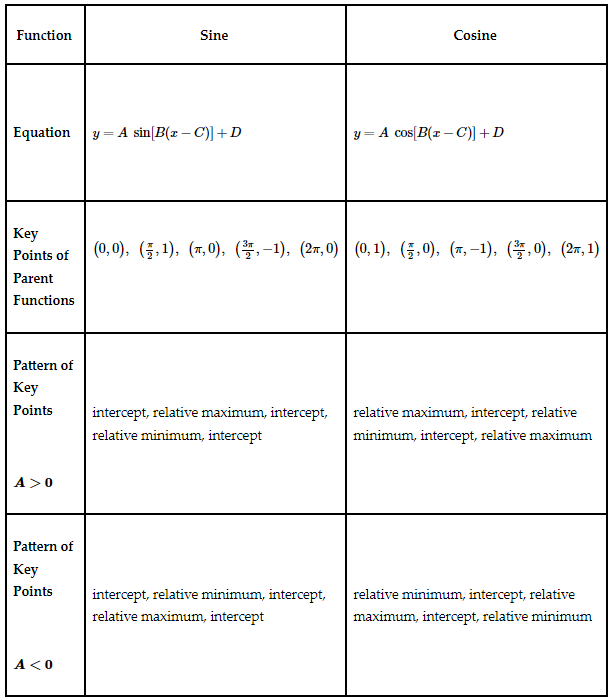
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| --- | --- |
| **Example:** Express as a sine function. | |
| **Step 1:** Find the period of the cosine function. | Recall that the period can be found by .  Period = |
| **Step 2:** Divide the period found of the cosine function by 4 to determine how many units the graph of is to be translated to the left. | Shifting the graph of by units to the left results in . |
| **Step 3:** Apply other transformations so that the two functions have the same graph. |  |
| **Step 4:** State the answer. | The graph of expressed as a sine function is . These two functions will result in the exact same graph. |

**Objective 3:** In this section, you will write equations as a sine or cosine function, given graphs with multiple transformations.

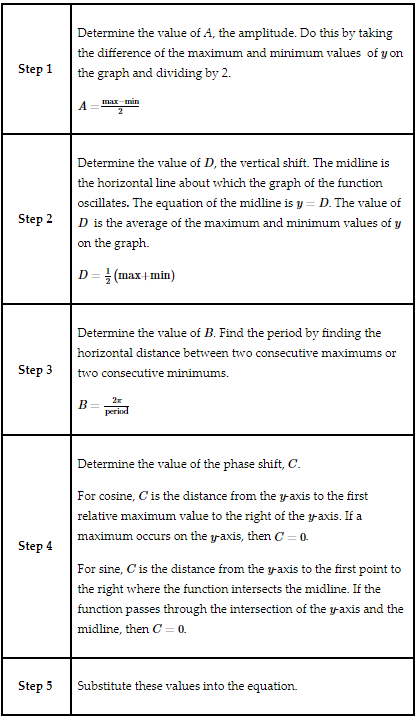
*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* You can use the following characteristics of the sine and cosine functions to write equations.



* You will use the characteristics from the table along with the [parameters of the functions](#Bookmark10) to determine an equation for a function given a graph.
* [Recall](#Bookmark11) how to rewrite sine and cosine functions as the opposite function.



|  |  |
| --- | --- |
| **Example:** Given the following graph, write an equation for a sine function and then a cosine function. | |
| **Write the given graph as a sine function first.**   |  |  | | --- | --- | | **Step 1** |  | | **Step 2** |  | | **Step 3** |  | | **Step 4** |  | | **Step 5** |  | | |
| **Write the equation as a cosine function.** | |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Jasmine is converting  to a cosine function.  Her first step was to rewrite the function as .  What transformation does she need to apply?  **Option #1:**Jasmine needs to shift the graph an additional  units to the right to obtain the cosine function .  **Option #2:**Jasmine needs to shift the graph an additional  units to the left to obtain the cosine function .  **Option #3:** Jasmine needs to shift the graph an additional  units to the right to obtain the cosine function .  Option #\_\_\_\_ describes the needed transformation. | 3 |
| P 2 | Rewrite the sine function  as a cosine function.  Option #1:  Option #2:  Option #\_\_\_\_\_ is correct. | 1 |
| P 3 | Eduardo rewrote the cosine function  as . What was his mistake?  Statement #1: Eduardo should have multiplied the period of 2π by ,  then added  to get  Statement #2: Eduardo forgot to factor out the coefficient  to get .  Next, he should add the shift of 2π to get  Statement #\_\_\_\_describes Eduardo’s mistake. | 2 |
| P 4 | Use the image to answer the question.  Using the given graph, write an equation for a sine function.  𝑦 = \_\_\_\_\_ |  |
| P 5 | Use the image to answer the question.  Using the given graph, write an equation for a cosine function.  *y* = |  |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Rewrite the sine function  as a cosine function. |  |
| Q 2 | Determine the period of the function . |  |
| Q 3 | Rewrite the cosine function  as a translated sine function. | ) |
| Q 4 | Use the image to answer the question.  Which of the following options correctly writes an equation for a cosine function, based on the given graph? |  |
| Q 5 | How do you write an equation as a sine function with four parameters – *A, B, C*, and *D* where *A* represents the amplitude, *B* affects the period of the function, *C* determines the horizontal shift (phase shift), and *D* represents the vertical shift? |  |