Algebra 1

**Functions & Their Graphs**

**Unit Summary:** By the end of this unit, you will be able to identify examples of functions you encounter every day. You will be able to demonstrate and recognize a function from equations, tables, and graphs.

**GeoGebra:** [Function Machine](https://www.geogebra.org/m/mzxvxe44), [Functions Activities](https://www.geogebra.org/math/functions)

**Lesson 2 – Relations & Functions**

**Key Words:**

* **domain** – the set of all possible inputs (x-values) of a function
* **equation** – a rule of correspondence between two quantities
* **function** – an expression, rule, or law that defines a relationship between the independent variable and the dependent variable
* **graph** – a visual diagram that shows the relationship between two quantities
* **mapping diagram** – a visual display that uses lines or arrows to show how an input value is paired with its corresponding output value
* **ordered pair** – two values written in a specific order, where x is the first value and y is the second value to be substituted for the variables in an equation; written as (x,y)
* **range** – the set of all outputs (y-values) of a function
* **relation** – a correspondence between two quantities
* **Vertical Line Test** – an evaluation used to determine whether a relation represented by a graph is a function by drawing vertical lines across the graph; if any of these vertical lines intersect the graph more than once, then the graph is not a function

**Objective 1:** In this section, you willexplain that a function is a specific type of relation that assigns each element of one set (the domain) to exactly one element of another set (the range).

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas**:

* A *relation* compares two sets of information which can be written as an *ordered pair*.
  + *Relations* can be represented using a table or a *mapping diagram*.
  + A relation has two components: the *domain* and the *range*.
    - The domain is the set of all input values, or *x-*coordinates.
    - The range is the set of all output values, or *y*-coordinates.
* For example, the relation below shows the relationship between age and height of high school students. It is represented in several different ways.

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| **Relations** | |
| **Table** |  |
| **Mapping Diagram** |  |
| **Ordered Pairs** |  |
| **Domain & Range** | **Domain:**  **Range:** |

* A *relation* that assigns each element of one set (the *domain*) to exactly one element of another set (the *range*) is called a *function*.
* For a *relation* to be a *function*, each input value must be assigned to no more than one output value.
  + If a *relation* is a *function*, no element of the *domain* will repeat.

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| **Relation (Not a function)** | **Relation (Is a Function)** |
| * The age 15 corresponds to two different heights (1.68 and 1.73). * You cannot say that each input (age) is assigned to exactly one output (height). * This relation is NOT a function. | * Each month is uniquely assigned to a single average lowest temperature. * The relation is a function. |

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| **Example:** Is the relation a function? Explain your answer. Then, find the domain and range. | |
| **Step 1:** Analyze the domain (*x-*values). | * Recall that a function will not have elements repeat in the domain. * In this relation, the element –3 repeats or corresponds to two elements of the range. * This relation is **not** a function. |
| **Step 2:** Write the domain and range. | * Domain (*x-*values): * Range (*y-*values):   \*Notice that we list the domain and range in numerical order and do not repeat values. |

**Objective 2:** In this section, you will identify function and non-function relationships in a variety of representations.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas:**

* Recall that a *function* is a specific type of *relation* that shows that for every input, there is exactly one corresponding output.
  + All *functions* are *relations*, but not all *relations* are *functions*.
* There are multiple ways to represent *relations* and *functions*:

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| **Representation** | **Example** |
| **Sets of Ordered Pairs**   * Relations can be written as a set of ordered pairs in the form (input, output), or . * Brackets {} are used to indicate that each of the ordered pairs within the brackets belong to the same set. | U.S. Cities and their zip codes:  {(Anchorage,99501), (Miami, 33162), (New York City, 10011), (New York City, 10007)}  \*This relation is not a function because New York City has two different outputs. |
| **Table of Values**   * The first column represents the *x* or input. * The second column represents *y* or output. | |  |  | | --- | --- | | **Letter Grade** | **Number Grade** | | A | 90-100 | | B | 80-89 | | C | 70-79 | | D | 60-69 | | F | Below 60 |   \*This relation is a function because each input is assigned to exactly one output. |
| **Mapping Diagram**   * Uses lines and arrows to show how an input value is mapped or paired with a corresponding output value. * The elements of the domain are listed on the left. * The elements of the range are listed on the right. | \*This relation is not a function because Alisha and Morgan are each in two different clubs making them have more than one unique output value. |
| **Graph**   * A graph is a set of ordered pairs plotted on the coordinate plane. * Graphs are used to determine the type of relationship that exists between the given quantities in a relation. |  |
| **Equation**   * If a relation is a function, you can say that y is a function of x. * This indicates that the output, *y*, depends on the input, *x*. * To show that *y* is a function of *x*, use the notation . | , where *y* represents the cost of renting a truck from a moving company and *x* represents the number of miles driven. |

* To determine whether a graph represents a *function* or non-function relationship, you can use an evaluation called the *Vertical Line Test*.
* The *Vertical Line Test* is performed by drawing vertical lines on any part of the graph. If any of these vertical lines intersect the graph more than once, then the *relation* represented is not a *function*.
  + In the examples below, the blue lines represent the *Vertical Line Test*.

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| **Function** | **Non-Function** |
| None of the vertical lines intersect the graph (orange line) more than once. | The vertical lines intersect the graph more than once. |

* Sometimes, you may be given an equation to graph to determine if it is a *function*. Follow the steps in the example below.

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| **Example:** Determine if is a function by graphing. | |
| **Step 1:** Substitute the values –2, -1, 0, 1, 2 into the equation for *x* and solve for *y*. |  |
| **Step 2:** Create a table of values for the inputs (*x*) and outputs (*y*). |  |
| **Step 3:** Graph the ordered pairs in a coordinate plane. |  |
| **Step 4:** State the answer. | Yes, this is a function because the graph of the equation passes the Vertical Line Test. |

**Lesson 4 – Naming, Evaluating, & Interpreting Functions**

**Key Words:**

* **dependent variable** – a mathematical variable (often represented by y) whose value is determined by one or more other variables in a function
* **evaluating a function** – the process of finding an output that corresponds to a given input
* **function notation** – a way to express the functional relationship between independent and dependent variables through symbols and is denoted as y = f(x) with x as the input value
* **independent variable** – a mathematical variable (often represented by x) that is independent of the other variables in an expression or function and whose value determines one or more of the values of the other variables

**Objective 1:** In this section, you will use function notation to describe a functional relationship between two quantities.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas**:

* *Function notation* is a way to express the relationship between *independent* (*x*-values) and *dependent* (*y*-values) variables.
  + denotes that *x* is a function of *y*.
  + The notation means exactly the same thing as *y*.
  + The name of the function in this case is , however, the name of the function can be any letter such as , and so on.

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| **Example:** The notation *h(L)* represents an adult female’s height for any femur length *L*.   * What is the name of the function? * Identify the independent and dependent variables. |
| * The name of the function is *h*. * The independent variable is the femur length, *L*. * The dependent variable is the adult female’s height, *h(L)*. |

* All functional relationships can be written in *function notation*.

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| **Relationship** | **Function Notation** |
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|  | The value of *x* is the exact same as the value of *y*. |

* You can also represent tables as function notation.

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| **Example:** The following function is represented in a table. Describe the relationship using function notation. | |
| **Step 1:** Observe the relationship between the input and output. | Each output is 5 more than the input. Thus, to find the output, add 5 to each input. |
| **Step 2:** Write in function notation. |  |

**Objective 2:** In this section, you will evaluate functions for inputs in their domains.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas:**

* The process of finding an output for a given input is called *evaluating a function*.
* Think of a function as a machine. When an input goes inside a function machine, it undergoes an operation, or a series of operations, and an output is produced.
* When you evaluate a function you are looking for the value of that corresponds to a given value of *x*.

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| **Function Machine** | |
|  | * The rule tells us to multiply the input by 3 and then subtract 7. Say the input is *x*. * If you multiply *x* by 3, then subtract 7 the result is . * The function that follows this rule is . |
| The function machine uses the input of 5, which means you must evaluate function at , as follows: | |

* To evaluate a function is to **substitute** the indicated value of the input to find the value of the corresponding output.

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| **Example:** For the function find . | |
| **Step 1:** Substitute the input for *x*. |  |
| **Step 2:** Simplify. |  |
| **Step 3:** State the answer. | For the function , when the input is –6, the output is 21. |

**Objective 3:** In this section, you will interpret statements using function notation in terms of a context.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas:**

* Recall that a function shows how one quantity is dependent on the other quantity by describing the relationship using *function notation*.
  + Recall that you must first name the function.
  + The variable that represents the independent value, or input, should be enclosed inside parentheses. For example, *x* is the input in .

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| **Example** |
| **Interpret:** Look at the function notation for the volume of a sphere and interpret.   * is the name of the function. * represents the volume of a sphere given a specific radius, *r*. * is the input value, or the value of the radius in this case. |
| **Evaluate:** You can evaluate this function for different values of using substitution.   * What is the volume of a sphere if the radius is 1?   + The input is 1, the output is . When the radius of a sphere is 1 unit, the volume of a sphere is cubic units. * What is the volume of a sphere if the radius is 3?   + The input is 3, the output is . When the radius of a sphere is 3 units, the volume of a sphere is cubic units. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Use the table to answer the question.   |  |  | | --- | --- | | ***x*** | ***f(x)*** | | -1 | -2 | | 0 | 2 | | 1 | 6 | | 2 | 10 |     Using function notation, what is the equation described in the table? | 4; 2 |
| P 2 | Use the image to answer the question.  This map shows ordered pairs that belong to the function  What is the missing value? | 2 |
| P 3 | What number would be output when an input of 4 is evaluated using the rule shown in the image? Enter your response in the output box. | 13 |
| P 4 | The number of bugs doubles every 4 days. If the bug population starts with 6 bugs, then the equation can represent the number of bugs. Find the value for  The number of bugs after 8 days is \_\_\_\_\_. | 24 |
| P 5 | A certain taxi company charges a flat fee of $15 per trip, plus $0.45 per mile. The function represents this cost. Find the cost of a 13-mile taxi ride.  The cost of the taxi ride is \_\_\_\_. | 20.85 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | *Use the table to answer the question*.   |  |  | | --- | --- | | ***x*** | ***f(x)*** | | 0 | −5 | | 1 | −9 | | 2 | −13 | | 3 | −17 | |  |
| Q 2 | Which equation describes the function shown in the mapping diagram?  Picture 1, Picture |  |
| Q 3 | Evaluate the function for the following input: . Which input generates the largest output when evaluating the function? |  |
| Q 4 | The function  represents the volume of a cylinder that is 10 inches high with a radius of r. Which is the correct interpretation for the function notation ? | The volume of the cylinder is 90π when the radius is 3. |
| Q 5 | Monique is punting a football and the height the ball travels is a function of time. The function  gives the height of the ball, , given the number of seconds, s, that the ball has been in the air. After flying through the air for 7.5 seconds, what is the height of the ball? Find | 21.75 |

**Lesson 5 – Representing Functions**

**Key Words:**

* **domain** – the set of all possible inputs (x-values) of a function
* **function** – an expression, rule, or law that defines a relationship between the independent variable and the dependent variable
* **graph** – a visual diagram that shows the relationship between two quantities
* **input-output table** – a table that shows the correspondence between the input and output of a function
* **range** – the set of all outputs (y-values) of a function
* **relation** – a correspondence between two quantities
* **Vertical Line Test** – an evaluation used to determine whether a relation represented by a graph is a function by drawing vertical lines across the graph; if any of these vertical lines intersect the graph more than once, then the graph is not a function

**Objective 1:** In this section, you will create input-output tables for a variety of function equations.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas**:

* Once you have a *function* described by a rule or equation, you can evaluate the *function* at any value of the input, and you can organize the input-output pairing by using *input-output tables*.

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| **Example 1:** Create an input-output table for with *x-*values of –2, -1, 0, 1, and 2. | |
| **Step 1:** Evaluate the function at the indicated values of *x*. [Recall](#Bookmark1) that you can use substitution to evaluate a function. | The input is and the output is . |
| **Step 2:** Enter each input-output pair into a table. [Recall](#Bookmark2) that the input values go in the left column and the output values in the right. | |  |  | | --- | --- | | **Input,** | **Output,** | |  |  | |  |  | |  |  | |  |  | |  |  | |

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| **Example 2:** A function represents the amount of medication, in milligrams, that remains in the body hours after it is taken. Complete the input-output table to determine the amount of medication remaining in the body at 0, 1, 2, and 3 hours after it is taken. | |
| **Step 1:** Evaluate the function at the indicated values of *t*. [Recall](#Bookmark1) that you can use substitution to evaluate a function. | The input is and the output is . |
| **Step 2:** Enter each input-output pair into a table. [Recall](#Bookmark2) that the input values go in the left column and the output values in the right. | |  |  | | --- | --- | | **Input,** | **Output,** | | 0 | 100 | | 1 | 50 | | 2 | 25 | | 3 | 12.5 | |

**Objective 2:** In this section, you will use input-output tables to create graphs of functions.

*Mathematical Practice Standard: Reason abstractly and quantitatively*.

**Big Ideas:**

* If you are given a *function* [represented by an equation](#Bookmark3), you can create an input-output table by evaluating the *function* at any values of the input.
* Once you have an input-output table, you can create an ordered pair that can be *graphed*.
  + Each input-output pair can be written as an ordered pair , where the input is the first coordinate, and the output is the second coordinate.

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| **Example:** Consider the function . Create an input-output table and graph the function. Use the input values . | |
| **Step 1:** Use substitution to evaluate the function for each input. |  |
| **Step 2:** Create an input-output table for the function. | |  |  | | --- | --- | | **Input,** | **Output,** | | -2 | 8 | | -1 | 5 | | 0 | 2 | | 1 | -1 | | 2 | -4 | |
| **Step 3:** Write each input-output pair as an ordered pair, or coordinate, . | |  |  |  | | --- | --- | --- | | **Input,** | **Output,** | **Ordered Pair,** | | -2 | 8 | (-2, 8) | | -1 | 5 | (-1, 5) | | 0 | 2 | (0, 2) | | 1 | -1 | (1, -1) | | 2 | 4 | (2, 4) | |
| **Step 4:** Graph the function. Plot each ordered pair and connect them with a line. |  |

* [Recall](#Bookmark4) that, for a *graph* to be a *function*, it must pass the *Vertical Line Test*.
* You can also use the *graph* of a *function* to find an output value for a given input value.

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| **Example:** Use the graph to find . | |
| **Step 1:** Identify the **input** from the given problem. | Recall that in function notation is the input and is the output.  So, in this problem is the output and is the input. |
| **Step 2:** Use the graph to find the output when . | To find , find a point on the graph with an *x-*value of 2. Then locate its corresponding *y*-value. |
| **Step 3:** State the answer. | This notation corresponds to the point on the graph (2, -4). |

**Objective 3:** In this section, you will relate verbal, numeric, algebraic, and graphical representations of functions to each other.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* Recall that a *function* is a *relation* between two sets of variables where one depends on the other. We refer to these variables in different ways sometimes.
  + is the independent variable, input value, or the *domain*.
  + is the dependent variable, output value, or the *range*.
* The *range*, or output, is dependent on the *domain*, or input.
* *Functions* can be represented in four different ways:
  + Algebraic: equation or formula
  + Numerical: table
  + Verbal: written description, often in real-world context
  + Graphic: visual representation on the coordinate plane

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| **Example:** Represent the following situation as a function in the various forms: verbal, algebraic, numerical, and graphic.  Ryan borrowed $250 from her father to purchase a new pair of wireless earbuds. She agreed to pay him back $50 a month. | |
| **Verbal** | The problem statement is already a verbal description of the function. |
| **Algebraic** | Let *x* represent the number of months that have passed. Let *y* represent the amount of money Ryan owes.  The equation that represents Ryan’s borrowing $250 from her father and paying him back $50 per month is which can also be written in function notation . |
| **Numerical** | To complete the numerical representation of the function, the equation , will be used to complete the table. |
| **Graphical** | The points generated from the table can be used to create the visual or graphic representation.  The graph shows the beginning balance of $250 and how much Ryan owes as each month passes. For example, after 3 months Ryan will owe $100, which is represented by the ordered pair (3, 100). |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Use the table to answer the question.    A car decreases in value after a given number of years. The following function represents the value of the car in dollars with respect to its age, *x*, in years: . Use the input-output table to determine the value of the car after 5 years. Round your answer to the nearest cent. | 15, 529.69 |
| P 2 | Use the image to answer the question.  Use an input-output table to determine whether the graph accurately displays the function  using the inputs of −3, −2,−1, 0, and 1.  Enter 1 if the graph is accurate.  Enter 2 if the graph is not accurate. | 1 |
| P 3 | Use the image to answer the question.  Use an input-output table to determine whether the graph accurately displays the function  using the inputs of −20, −10, 0, 10, and 20.  Enter 1 if the graph is accurate.  Enter 2 if the graph is not accurate. | 2 |
| P 4 | *Use the image to answer the question.*  Based on the graph of the function, what value will correctly complete the sentence?  Josie can run two miles in \_\_\_\_ minutes. | 12 |
| P 5 | Use the image to answer the question.  Luis rents a paddleboat to use at West Park for the day. He has to pay a deposit and an hourly fee. Using information from the graph, enter the values that complete the equation for this situation.  *y* = \_\_\_\_ *x* + \_\_\_\_\_ | 5; 20 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Use the table to answer the question.    Finish creating the input-output table for the function . Which of the given x values produces the largest output for the function? | *x* = -1 |
| Q 2 | Use input-output tables to determine which graph accurately represents the function  with inputs of −4, −2, 0, 2, and 4. | Five points are plotted on a coordinate plane, and a line is graphed passing through them. The x-axis ranges from negative 4 to 4 in increments of 1. The y-axis ranges from negative 10 to 10 in increments of 1. |
| Q 3 | Ari is starting a yard service business. He charges a flat rate of $10 plus an additional $15 per hour, regardless of the service. Create a function and use a table to determine how much Ari will make if he works for 1, 1.5, 2, 2.5, and 3 hours. | (1,25), (1.5,32.5), (2,40), (2.5,47.5), (3,55) |
| Q 4 | Noah has to read a book for a project. They have already read 105 pages, and they can read about 32 pages per day. Which of the following functions correctly relates the verbal information about the function to an algebraic equation? |  |
| Q 5 | *Use the image to answer the question*.  This function models the number of points a student scores on a 10-point quiz as a function of the hours she spent playing video games the night before. Which of the following statements is correct? | The student’s quiz score goes down by one point for every two hours she spent on video games. |

**Lesson 6 – Linear Functions from Situations**

**Key Words:**

* **dependent variable** – a mathematical variable (often represented by y) whose value is determined by one or more other variables in a function
* **function** – an expression, rule, or law that defines a relationship between the independent variable and the dependent variable
* **independent variable** – a mathematical variable (often represented by x) that is independent of the other variables in an expression or function and whose value determines one or more of the values of the other variables
* **linear function** – a function that represents a straight line on the graph where x’s do not repeat
* **origin** – a starting point on the coordinate plane (0,0), the point of intersection of the x and y axis
* **quantities** – the values of variables, independent (x) and dependent (y), that are used in tables and in plotting graphs
* **scale** – the distance between each square on the coordinate plane
* **slope** – the steepness of a line, found by dividing the change in the y-value by the change in the x-value
* **slope-intercept form** – the equation that represents linear relationships (y=mx+b), where m is the slope and b is the y-intercept
* **y-intercept** – the y-coordinate of a point where a line, curve, or surface intersects the y-axis

**Formulas:**

* Slope-Intercept Form:
* Slope Formula:

**Objective 1:** In this section, you will define appropriate quantities, origin and scale of graphs of linear functions from real-world situations.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* Graphs of *functions* can show changes in speed, altitude, distance, volume, time, and other variable quantities.
* *Linear functions* represent a straight line on the coordinate plane.
* To make sure the graph represents a given situation, you will need to consider the following elements:
  + Labeling of the graph.
    - The *independent variable* (input) is represented by the *x-*axis and the *dependent variable* (output) by the y-axis.
  + Scaling the graph.
    - The *scale* is the distance between two numbers that indicates a unit that must remain uniform across an axis.
    - The *scale* shows numerically how the *dependent variable* (*y*) changes in relation to the changes of the *independent variable* (*x*).
* When setting up graphs from situations, you should consider the parameters needed to display the information on a graph.
  + What are the *quantities* (dependent and independent variables) with which you will label the *x-* and *y-*axis?
  + In what numerical increments should the x-axis and y-axis be *scaled*?
  + Are all four quadrants of the coordinate plane needed?
  + What is the point of origin where the line should begin?

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| **Example:** Kennedy makes $15 an hour working at a local bookstore. From this situation, label a graph to show quantities, origin, and scale. | |
| **Step 1:** What are the *quantities* (dependent and independent variables) with which you will label the *x-* and *y-*axis? | Independent Variable (*x*): Hours Worked  Dependent Variable (*y*): Amount Paid  You would use “Hours Worked” to label the x-axis. You would use “Amount Paid” to label the y-axis.  These labels tell us that the number of hours worked will determine how much Kennedy gets paid. |
| **Step 2:** In what numerical increments should the x-axis and y-axis be *scaled*? | Make the scale for the *x-*axis in increments of 1 to indicate the number of hours worked.  Make the scale for the *y-*axis at increments of 5 to indicate the amount of money she is paid for the hours worked. |
| **Step 3:** Are all four quadrants of the coordinate plane needed? | This situation is confined to quadrant 1 because negative hours and negative pay are not viable for the solution. |
| **Step 4:** Identify the starting point or origin. | The starting point of the graph is (0,0) and shows that at zero hours worked, Kennedy has earned $0. |
| **Step 5:** Graph the situation. | The line indicates that the more hours Kennedy works, the more money she will get paid.  Inserting image... |

**Objective 2:** In this section, you will plot points to create linear functions from situations.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* Given a situation, such as a word problem, you can use the details to create a graph of its *linear function* and plot points along the graph.
* To graph a *linear function*, you must identify **two** points. After you have these two points, you can connect the points to form a line.
  + The easiest point to identify in a given situation is the *y-intercept*, which acts as the starting point of the line (when *x* is zero).
  + The second point is sometimes given in the text of the problem, but if it is not:
    - Use the *slope* or rate of change of the line which tells you how far one point is from another point.
* When a situation does not provide enough information to identify two points, you can write an equation that represents the *linear function* to generate points.
* Equations representing linear functions are typically written in *slope-intercept form*.

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| **Slope-Intercept Form** |
| * slope or rate of change * y-intercept * independent variable (input) * dependent variable (output) |

* To generate the points from the equation, you will substitute *x-*values into the equation to solve for the *y-*values.
* These values create the ordered pairs that are then plotted on the graph to create a line that represents the *linear function*.

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| **Example:** An elephant calf weighs 282 pounds at birth. The calf gains 2.5 pounds each day for its first year of life. After 1 year, the calf weighs 1,195 pounds.   * Identify and graph two points that can be used to plot a graph to represent this linear function. * The equation that represents this linear function is . Round the weights to the nearest whole number. | |
| **Step 1:** Evaluate the function at various days to create an input-output table. | Inserting image... |
| **Step 2:** Generate a set of points that can be plotted from the input-output table. |
| **Step 3:** Plot the points on a graph and connect them with a line. | Inserting image... |

**Objective 3:** In this section, you will interpret key features of linear function graphs in context.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* In addition to labels on the x-axis and the y-axis, two other features are essential to interpreting a graph of a function.
  + *Intercepts* are the points where the line created by the function crosses the *x-*axis or the *y-*axis. The intercepts help us understand where the story begins or ends.
    - *y-intercept* is where the input, or *x*-value, is zero.
    - *x-intercept* is where the output, or *y*-value, is zero.
  + The *slope* of the line tells us what is happening throughout the story.
    - The *slope's* intervals describe whether the function increases or decreases and at what rate.
    - *Slope* is calculated by dividing the change in *y* over the change in *x*.
    - Recall that in *slope-intercept form* () the slope is represented by the variable .

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| **Slope Formula** |
| When given two points on the graph of a function and . |

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| **Example:** The following graph represents the amount of money earned over several hours. Interpret the graph by answering the following questions.   * What is the *y-*intercept, and what does it mean in context of the problem? * What is the slope of the line, and what does it mean in the context of the situation? * Although the point is not represented on the graph, what will the total be if ? And what will the total mean? | |
| **Step 1:** What is the *y-*intercept, and what does it mean in context of the problem? | * The line crosses the y-axis at (0,0) which is the y-intercept. * Using the labels to interpret the context, this intercept means that at 0 hours worked, $0 dollars will be paid. |
| **Step 2:** What is the slope of the line, and what does it mean in the context of the situation? | Use the slope formula to calculate the slope:   * Pick any two points that fall on the graphed line. For example, (0, 0) and (2, 60).      * and * The slope of the line is 30, meaning this person earns $30 for every hour worked. |
| **Step 3:** Although the point is not represented on the graph, what will the total be if ? And what will the total mean? | If we know that $30 is earned after each hour of work, then after 15 hours of work $450 dollars will be earned. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Use the image to answer the question.  Lina charges a one-time fee of $5, plus $10 per hour, for a dog-walking appointment. Does the current graph use a reasonable scale for this situation? Enter 1 for yes or 2 for no. | 2 |
| P 2 | Use the image to answer the question.  Sal is draining his 25-gallon fish tank at approximately half a gallon per minute and graphed the situation. Is the scale reasonable for this graph? Enter 1 for yes or 2 for no. | 1 |
| P 3 | The cost of taking a cab is $4 plus $0.75 per mile. This can be represented using the linear function , where c is the cost per trip and m is the distance in miles. Graph the equation on a coordinate plane. Based on your line, what would the cost be if you were to travel 11 miles? Round your answer to the nearest hundredth. | 12.25 |
| P 4 | Use the image to answer the question.  Identify the y-intercept. | 6 |
| P 5 | Use the image to answer the question.  Identify the slope. | 3 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | A graph of the function  models Carli’s road trip, with her speed measured in miles per hour. The graph includes the two hours of stops she’ll make. Using this information, define which quantity describes the x-axis. | time in hours |
| Q 2 | Use the image to answer the question.  The graph models the descent of a hot air balloon in feet per minute. What does the y-intercept tell you in this situation? | The balloon starts its descent at 600 feet. |
| Q 3 | A rental truck company charges $25 to rent a truck, then $1.25 for every mile it is driven. The situation can be expressed as , where C is the total cost to rent the truck and m is the miles driven. If you plot points on the linear function, which of the following would appear? | (4,30) and (8,35) |
| Q 4 | Use the image to answer the question.  A hot air balloon climbs into the air at a steady rate per minute. The graph illustrates the hot air balloon’s ascent. Interpret the slope of the graph. | For every four minutes that pass, the balloon rises 200 feet. |
| Q 5 | Use the image to answer the question.  A line with an arrow at the bottom is plotted in quadrant 1 of a coordinate plane. The x-axis is labeled Months, and the y-axis is labeled Amount in Dollars. , Picture  This graph shows data about Barton’s banking situation. Which statement correctly interprets the slope of the line in the graph? | ; Barton pays $175 per month to pay off a loan. |

**Lesson 7 – Piecewise Linear Functions from Situations**

**Key Words:**

* **absolute value function** – a function that contains an algebraic expression and the use of absolute value bars
* **dependent variable** – a mathematical variable (often represented by y) whose value is determined by one or more other variables in a function
* **domain** – the set of all possible inputs (x-values) of a function
* **independent variable** – a mathematical variable (often represented by x) that is independent of the other variables in an expression or function and whose value determines one or more of the values of the other variables
* **interval** – a set of real numbers between two numbers either including or excluding one or both of them
* **linear function** – a function that represents a straight line on the graph where x’s do not repeat
* **piecewise function** – a group of two or more functions broken up into specific intervals over the domain
* **piecewise linear function** – a function which uses a combination of linear equations over the intervals of its domain
* **restricted function** – a mathematical expression, usually an interval, which limits the domain of a given function
* **step-function** – a mathematical function of a single real variable that remains constant within each of a series of adjacent intervals but changes in value from one interval to the next

**Objective 1:** In this section, you will define appropriate quantities, origin and scale for graphs of piecewise linear functions when given different situations.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

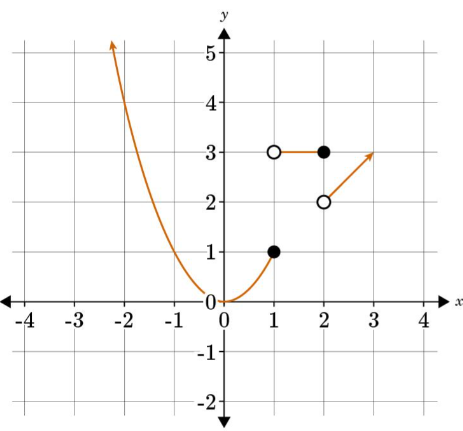
* *Linear functions* represent one relationship between two quantities, and *piecewise linear functions* represent a combination of linear functions.
* A *step function* is a type of *piecewise function* that defines situations that have constant rates between intervals.

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| **Example:** A college student decided to open a tutoring company and works at a **set rate of $25 per hour** or any portion of that hour. This means that they will charge $25 as soon as the first hour of tutoring begins, another $25 for the second hour, and so on. A parent wants to know what the cost would be if her son is tutored for **3.5 hours**.   * Describe the quantities, as well as the relationships between quantities and scales, that should be used in the graph to represent this scenario. | |
| * The quantities represented are time and cost. * Time in hours is the independent *x* variable. * Cost in dollars is the dependent *y* variable. |  |
| * The total cost for just 1 hour of tutoring is $25. Since these two numbers are not close, different scales are needed for the x- and y-axis. * Time can be represented in increments that increase by one. * Cost can be represented in increments that increase by $25. |

**Objective 2:** In this section, you will based on different situations, you will plot points to create graphs of piecewise linear functions.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* The process of graphing a *piecewise function* is very similar to that of graphing a normal function ([recall here](#Bookmark5)) with one main difference:
  + A *piecewise function* is made up of “pieces” from at least two functions. Each “piece” of the function occupies a different part of the *domain*.
  + Recall that the *domain* of a function is the set of all possible input, or *x-*values.
* For example, the following graph shows three pieces of the function that occupy a specific part of the graph. Each piece of the graph is a *restricted function*.
  + 
* *Restricted functions* create *piecewise functions* because the *domain* is limited using *intervals*.
* Tables are useful for graphing functions with a given interval.
  + You can create an input-output table with the values specifically defined, or *restricted*, by the interval.
* It’s important to pause here and recall the correct graphing notations for inequalities.
  + An open circle is used for inequality symbols:
  + A closed (or solid) circle is used for inequality symbols:

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| **Example:** Consider the function with a domain of . Graph the function. | |
| **Step 1:** Interpret the domain. | The interval tells you that when evaluating this function, the only valid input values of *x* are those that are **greater than 2**.  However, the input value of 2 must also be included in the table and must be evaluated because it is a boundary point for this function |
| **Step 2:** Evaluate the function in an input-output table. It’s best to evaluate 3-4 inputs. | Notice how all the input values are greater than 2.   |  |  | | --- | --- | |  |  | | 2 |  | | 3 |  | | 4 |  | | 5 |  | |
| **Step 3:** Create ordered pairs for the input-output table and indicate the type of circle (open or closed). | |  |  |  |  | | --- | --- | --- | --- | | **Input** | **Output** | **Ordered Pair** | **Circle** | | 2 | 4 | (2,4) | open | | 3 | 5 | (3,5) | closed | | 4 | 6 | (4,6) | closed | | 5 | 7 | (5,7) | closed |   Note that the open circle on point (2,4) because the input value is a boundary point for the function but is not included in the solution set “greater than 2”. |
| **Step 4:** Plot the points and graph the function. |  |

* Sometimes the intervals are compound inequalities like in the following example.

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| Consider the function restricted on the interval . | * Notice that the graph terminates on both the left and right side of the graph at –2 and 2 on the *x*-axis. * The open circle on (-2,6) indicates an interval restriction where the input –2 is not included in the solution set. * The closed circle on (2,6) indicates that the input value of 2 is included in the solution set. |

* Other times, these functions will be combined in one graph to create a piecewise function. Take the two functions graphed above and see how they can be combined.

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| **Combined Functions** | |
| Take the two functions from the examples above and combine them using proper notation: |  |

**Objective 3:** In this section, you will interpret key features of piecewise linear function graphs in context.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* *Piecewise linear functions* can also be used to represent real-world situations.

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| **Example:** Ships Ahoy charges $25 per person for a chartered cruise for up to 20 people. To encourage larger parties to book a cruise of more than 20 people and up to 40 people, the cruise company charges a flat rate of $500.  The scenario is modeled in the following graph. | |
| **Step 1:** Define the variables. | * Let *n* represent the number of people who go on the cruise. * Let *c* represent the total cost of the cruise**.** |
| **Step 2:** Create the equations and intervals that represent each situation. | * For groups of 20 people or less, the cruise charges $25 per person.   + Interval: * For groups of more than 20 and up to 40 people the cruise charges a flat rate of $500.   + Interval: |
| **Step 3:** Write the situation in proper piecewise function form. |  |

* A *step function* is a mathematical function of a single real variable that remains constant within each of a series of adjacent intervals but changes in value from one interval to the next.

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| **Example:** The shipping fees for Rustica Country Store’s online orders depend on the total cost of the items ordered. Graph the situation. | |
| **Step 1:** Define the variables. | * Let *c* represent the total cost of items ordered. * Let *f* represent the shipping fee. |
| **Step 2:** Define the four equations and their intervals. | * for * for * for * for   This is notated as the following: |
| **Step 3:** Graph each equation on the same graph. |  |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | If you earned up to $120,000 in 2011, you were taxed at a rate of roughly 6% of your income. If you made over $120,000, you were taxed at a flat rate of $8,000. If you were to graph this as a piecewise function to represent the tax for incomes between $0 and $750,000, what quantity would be represented on the y-axis?  Type 1 for salary.  Type 2 for amount of tax. | 2 |
| P 2 | *Use the image to answer the question*.  An online clothing company that does not offer any free items, charges a shipping fee of $8 for orders under $25, $4 for orders between $25 and $49.99, and no shipping fee for orders of $50 or more. The following piecewise function shows total cost for a shipment.  Does the graph accurately represent the function? Enter 1 for yes or 2 for no. | 1 |
| P 3 | Use the image to answer the question.  A math club wants to buy t-shirts for its members. The printing company has tiered pricing depending on how many shirts are purchased, with 𝑓(𝑥) representing the total cost of the shirts depending on how many are purchased.  Does the graph accurately represent the piecewise function? Enter 1 for yes or 2 for no. | 2 |
| P 4 | Use the image to answer the question.  A piecewise linear function consisting of three parts is plotted in quadrant 1 of a coordinate plane. The x-axis is labeled Time in minutes, and the y-axis is labeled Distance in miles.  , Picture  Christian covered a total distance of 1.5 miles while exercising. Sometimes he was running, sometimes he was jogging, and sometimes he was resting. During which time interval was Christian resting?  Christian rested from \_\_\_\_\_\_\_\_\_. | 12; 16 |
| P 5 | *Use the image to answer the question.*  Based on the graph, what will the price be for six customers?  The price for six customers is $\_\_\_\_\_. | 90 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | A dog boarding facility charges daily based on a dog’s weight. If the dog weighs under 15 lbs, the rate is $30. If the dog is between 15 and 50 lbs, it is $35. If the dog is 50lbs or more, the charge is $35 plus $3 for each additional 5 lbs over 50 lbs. Which of the following **best** defines the quantity and scale for the x-axis for a piecewise linear graph of the fee structure? | weight; 10 |
| Q 2 | A gas station is offering a deal. If you buy 10 gallons or more of gas, you will be charged $2.25 per gallon instead of the regular $2.50 per gallon. The deal is modeled by the following piecewise function.  If you were to create a graph of this piecewise function, which point would have an open circle? | (10,25) |
| Q 3 | Use the image to answer the question.  Picture 1, Picture  Children’s cough syrup has dosage guidelines based on a child’s age. In the following piecewise function, x represents the child’s age in years and  represents the dosage in fluid ounces.  Does the graph accurately represent the piecewise function? Why or why not? | No. There should be an open circle on the first point of two parts of the function. |
| Q 4 | Use the image to answer the question.  Interpret the key function of the graph for the domain interval . What scenario can be represented by this portion of the piecewise function? | a runner resting for 5 minutes |
| Q 5 | Use the image to answer the question.  The cost for shipping by the Boston Mailing Company is based on the amount that a customer spends. For customers spending under $50, the shipping rate is $10; for $50 to under $100, the shipping rate is $7; for $100 to under $140, the shipping rate is $4; for $140 and over the shipping rate is $2. What domain interval will have a shipping rate of $4? |  |

**Lesson 8 – Exponential Functions from Situations**

**Key Words:**

* **depreciation** – a decrease in the value of an item over time
* **domain** – the set of all possible inputs (*x*-values) of a function
* **exponential function** – a function in the form , where a is the initial value and b is the multiplier
* **range** – the set of all outputs (*y*-values) of a function

**Formulas:**

* Exponential Function:
  + Growth Factor:
  + Decay Factor**:**

**Objective 1:** In this section, you will define appropriate quantities, origin, and scale for graphs of exponential functions from situations.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* Exponential functions are used to model growth and decay such as bacteria, population, and compound interest.

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| **Exponential Function** |
| * is the initial value * is the multiplier or growth/decay factor   + The multiplier is calculated from the rate, .   + Growth:   + Decay: |

* Exponential functions are used to model growth and decay, such as bacteria, population, and compound interest.

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| **Example:** Suppose Aisha invests **$5,000** in an account that earns **4%** interest compounded annually.   * Find the value of Aisha’s account each year for **20 years**. * Make a graph to model the value of Aisha’s account each year. | |
| **Step 1:** Identify the initial value,. | Aisha initially invested $5,000. |
| **Step 2:** Calculate the multiplier, . | The value of the account increases at a rate of 4% each year, which is .04 in decimal form.  The multiplier, , is 1.04. |
| **Step 3:** Create an exponential function using the form . | where *x* is the number of years and *f(x)* is the value of the account |
| **Step 4:** Determine the value of the account after 20 years. | After 20 years Aisha’s account has a value of about $10,956. |
| **Step 5:** Determine a reasonable domain and range for the scale of the graph. | Because we only want to find the value of Aisha’s account over a 20-year period, the domain (input/*x*-values) only needs to go up to 20.  Domain:  Aisha’s account starts at $5,000 and will increase to about $11,000 over 20 years.  Range:  You will use these parameters for the domain and range when graphing the function. |
| **Step 6:** Determine an appropriate scale for each axis before graphing. | The *x-*axis represents the number of years which are measured in increments of 1.  The *y-*axis represents the value of the account which starts at $5,000 and will increase, an appropriate scale can be increments of $500. |
| **Step 7:** Graph the equation using technology like GeoGebra. |  |

**Objective 2:** In this section, you will plot points to create graphs of exponential functions from situations.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* [Recall](#Bookmark6) that the form of an exponential function is .
* Graphing an *exponential function* follows a similar process as graphing a linear function.
* When you need to graph an *exponential function*, you can make a table of values using technology or calculate the values using the equation of the *exponential function*.

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| **Example:** A doctor is studying the growth of bacteria in a lab experiment. She puts **fifty** bacteria in a petri dish to start and observes that the population **doubles in size every hour**. Model and graph the function where *h* represents the number of hours and *b* represents the number of bacteria. | |
| **Step 1:** Model the situation in the form of an exponential function. | * Initial value: * Multiplier: |
| **Step 2:** Make a table of values by calculating the values using the equation. | |  |  |  | | --- | --- | --- | | ***h*** | **Calculation:** | ***b(h)*** | | 0 |  | 50 | | 1 |  | 100 | | 2 |  | 200 | | 3 |  | 400 | | 4 |  | 800 | | 5 |  | 1,600 | |
| **Step 3:** Create ordered pairs from the table in step 2. | |  |  |  | | --- | --- | --- | | **h** | **b(h)** | **Ordered Pair** | | 0 | 50 | (0, 50) | | 1 | 100 | (1, 100) | | 2 | 200 | (2, 200) | | 3 | 400 | (3, 400) | | 4 | 800 | (4, 800) | | 5 | 1,600 | (5, 1600) | |
| **Step 4:** Plot the points on the graph and draw a curve through the points. |  |

**Objective 3:** In this section, you will interpret key features of exponential function graphs in context.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* The graph of an *exponential function* may model growth or decay.

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| **Key Features of Exponential Graphs** | |
| **Exponential Growth**   * As *x* increases, *y* increases * As *x* increases, *y* increases quickly at first and then slowly * From left to right, the graph is almost level at first and then has a steep increase | **Exponential Decay**   * As *x* increases, *y* decreases * As *x* increases, *y* decreases quickly at first and then slowly * From left to right, the graph has a steep decline at first and then almost levels off |
| Key features for both exponential growth and decay graphs are:   * The *y-*intercept, *a*, must be greater than 0 () * The *x-*values (domain) are from negative infinity to positive infinity (all real numbers) * The *y-*values (range) must be greater than 0 (). * The graph will never touch the *x-*axis | |

* [Revisit the example](#Bookmark7) from the previous objective about bacteria growth.

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| **Example:** The situation is modeled by the function , where represents the number of bacteria after hours, and represents the number of bacteria. Identify the key features of a graph. |
| * Examine the change in the values as increases. * The increases are getting larger and larger. That’s why the graph is getting steeper.      * As increases, increases. * The y-intercept, , is greater than 0. * The *h-*values (domain) include 0 and go to the hour that the experiment is stopped. * The -values (range) are greater than 0. The graph will never touch the x-axis. * The initial value is 50 and increases from there. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Karen purchased a car for $40,000 by taking out a loan that will take 7 years to pay off. Given the car depreciates in value by 11% each year, the situation can be modeled by the equation , where v is the value of the car after t years. If Karen wants to know how much the car will be worth in 7 years, which of the following options would be an appropriate range?  Option #1:  Option #2:  Option #3:  Option #4: | 2 |
| P 2 | Use the table to answer the question.    Find the missing value in the table for the exponential function . Round your answer to two decimal places.  = \_\_\_\_\_ | 6.22 |
| P 3 | *Use the table to answer the question*.    Cason is scheduled to get his wisdom teeth removed. The oral surgeon gives him a local anesthetic to numb his mouth before surgery. Cason is given 320 milligrams of the anesthetic, which metabolizes at a rate of 25% per hour. The situation can be modeled using the exponential equation . Finish the chart to determine which graph displays the correct plotted points for the situation.  Option #1:   Option #2:   Option #3: | 1 |
| P 4 | Use the image to answer the question.  Maria took 500 mg of medicine for her headache. The drug depletes in the blood stream at a rate of 20% per hour. The situation can be modeled by the exponential equation , where a is the amount of medicine in the blood stream after t hours. Which one of the following statements is true about the graph of the function?  Option #1: As t increases, a increases slowly at first and then quickly.  Option #2: The y-intercept of the function is (0,500).  Option #3: The range includes all real numbers. | 2 |
| P 5 | Use the image to answer the question.  Nate’s parents invested $2,000 in a savings account when he was born. If the account has a growth rate of 8% per year, the situation can be modeled by the equation , where a is the amount in the investment after t years. Which one of the following statements is true about the graph of the function?  Option #1: As t increases, a increases slowly at first and then quickly.  Option #2: The y-intercept of the function is (0,8).  Option #3: The range includes all real numbers. | 1 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Annabel wants to make banana bread for the bake sale. She went to the market to buy bananas, and she noticed fruit flies after she returned home with her produce. The number of fruit flies grows at an exponential rate modeled by the equation , where f is the number of fruit flies after t days. Define an appropriate domain for the problem if Annabel bakes the banana bread 5 days after returning from the market. |  |
| Q 2 | Callie entered an art contest in second grade and won a $1,000 scholarship. The money was invested in an account paying a 9% interest rate compounded annually. The situation can be modeled by the equation , where a is the amount in the account after t years. If Callie uses the scholarship 10 years later, determine which graph accurately displays the situation. | An increasing curve with an arrow at the end is plotted in the first quadrant of a coordinate plane. The x-axis ranges from 0 to 14 in increments of 2 and the y-axis ranges from 0 to 3,500 in increments of 500. |
| Q 3 | Use the table to answer the question.    A certain population of beetles is increasing at a rate of 22% per month. If there are currently 350 beetles, the equation  can be used to model the situation. Finish the chart and plot the points to determine which graph correctly displays the situation. | An increasing curve with an arrow on the top is plotted on a coordinate plane. The x-axis for months ranges from 0 to 10 in increments of 1. The y-axis for Beetles ranges from 0 to 2000 in increments of 250. |
| Q 4 | Use the image to answer the question.  The value of a cell phone decreases exponentially at a rate of 24% each year after its release date. If the initial value of the phone is $700, the situation can be modeled by the equation , where v is the value of the cell phone t years since its release date. Interpret the graph of the equation and identify the true statement. | As *t* increases, *v* decreases quickly at first and then slowly. |
| Q 5 | *Use the image to answer the question.*  Mason is completing a study for his psychology course. For the study, he begins with $20 and asks individuals on the street if they want the money or if they would like him to double it and give it to the next person. He continues to do this until someone takes the money. This situation can be modeled by the exponential equation , where *m* is the money and *p* is the number of people who opt to pass it on to the next person. Which of the following statements is true about the equation? | The range (*m*-values) includes the values |

**Lesson 9 – Quadratic Functions from Situations**

**Key Words:**

* **axis of symmetry of a parabola** – the straight line that divides a parabola into two identical parts
* **domain** – the set of all possible inputs (*x*-values) of a function
* **parabola** – a curve where any point is equidistant from a fixed point (the focus) and a fixed straight line (the directrix); the focus may not lie on the directrix
* **quadratic equation** – any equation containing one term in which the unknown is squared and no term in which it is raised to a higher power
* **quadratic formula** – a formula that gives the solutions of any quadratic equation in the form, , where .
* **range** – the set of all outputs (*y*-values) of a function
* **vertex of a parabola** – the highest or lowest point of a parabola that crosses its axis of symmetry
* ***x*-intercepts of a parabola** – the points where a parabola intersects the *x*-axis
* ***y*-intercept of a parabola** – the point where a parabola intersects the *y*-axis; the *x*-values that make the quadratic equation equal to zero; also call zeroes

**Formulas:**

* Quadratic Function:
* Quadratic Equation:

**Objective 2:** In this section, you will plot points to create graphs of quadratic functions from situations.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* *Quadratic formulas* are used to model many real-world situations.
  + A *quadratic equation* is in the form
  + The graph of a *quadratic* is a *parabola* and is shaped like an upward or downward U.

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| **Example:** Suppose a rock is dropped off a cliff that is 64 feet high. The equation that models this situation is where represents height and represents time. When will the rock hit the ground? | |
| **Step 1:** Make a table of values (points) for the parabola. | * Be sure to include the *x*-intercept(s), *y*-intercepts, and minimum/maximum in the table, when possible.  |  |  |  | | --- | --- | --- | | **Time *t*** | **Calculation** | **Height *h*** | | 0 |  | 64 | | 0.5 |  | 60 | | 1 |  | 48 | | 1.5 |  | 28 | | 2 |  | 0 | |
| **Step 2:** Create ordered pairs for the table in step 1. | |  |  |  | | --- | --- | --- | | **Time *t*** | **Height *h*** | **Ordered Pair** | | 0 | 64 | (0, 64) | | 0.5 | 60 | (0.5, 60) | | 1 | 48 | (1, 48) | | 1.5 | 28 | (1.5, 28) | | 2 | 0 | (2 ,0) | |
| **Step 3:** Plot the points on the coordinate plane and connect the points with a curved line. |  |
| **Step 4:** State the answer. | The rock will hit the ground in 2 seconds when and (representing a height of 0 feet). |

**Objective 3:** In this section, you will interpret key features of quadratic function graphs in context.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* Recall the general form of a quadratic equation is .
  + If *a* is positive, the parabola opens upward.
  + If *a* is negative, the parabola opens downward.
* The *vertex of a parabola* is the highest or lowest point of the parabola that crosses its *axis of symmetry*.
* The *axis of symmetry* is the straight line that divides a parabola into two identical parts.
* The key features of a parabola can be observed by looking at the graph.

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| **Key Features of a Parabola** | |
| **Upward Parabola** | **Downward Parabola** |
| * As *x* first increases, *y* decreases * It reaches a minimum, and then *y* increases | * As *x* first increases, *y* increases * It reaches a maximum, and then *y* decreases |
| * The *x-intercepts of a parabola* are the points where the parabola intersects *x*-axis. * The *y-intercept of a parabola* is the point where a parabola intersects the *y-*axis. | |

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| **Example:** Jon owns a jewelry-making business. His profit for selling necklaces is modeled by the equation , where *n* represents the price of a necklace, and *P* represents profit.   * Observe the graph and identify the key features. | |
| **Step 1:** Graph the quadratic equation using GeoGebra. | GeoGebra: [Key Features of a Parabola Example - GeoGebra](https://www.geogebra.org/calculator/ak8kknkc) |
| **Step 2:** Identify the *x-*intercepts. | The x-intercepts are at (10,0) and (50,0).   * The intercepts mean that if Jon charges $10 or $50 per necklace, then his profit will be $0. * If Jon charges between $20 and $50, then he will make a profit. |
| **Step 3:** Identify the axis of symmetry. | The axis of symmetry is .   * The axis of symmetry is in the middle of the *x-*intercepts, and the vertex lies on this line. |
| **Step 4:** Identify the vertex. | The vertex of (30, 4000) is the maximum of the parabola.   * The vertex tells us that if Jon charges $30 per necklace, then he will earn a maximum profit of $4,000. |
| **Step 5:** Identify the y –intercept. | The y-intercept is (0, -5000).   * This means that Jon has $5,000 in expenses before he sells any necklace. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | A football player kicks the ball with an initial upward velocity of 64 feet per second. This situation is modeled using the equation , where h is the height in feet and t is the time in seconds. Using this equation, what is the maximum height the ball will reach? | 64 |
| P 2 | Nora is creating a dog run for her dog, Mr. Darcey. She has enough fencing material for a fence with a perimeter of 120 feet. This situation is modeled using the equation  where 𝐴 is the area and w is the width. Using this equation, identify the maximum range or area the fence can have. | 900 |
| P 3 | Graph the function . What is the value of *f(x)* when . | 4 |
| P 4 | Graph the function . True or false: The x-intercepts of this graph are (0,0) and (1,0).  Type 1 for true.  Type 2 for false. | 1 |
| P 5 | The profit of a company is represented as , where 𝑃 represents the profit and x represents the amount spent on marketing. What is the dollar amount that should be spent on marketing to achieve the maximum profit? | 100 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | A volleyball is served by a 6-foot player at an initial upward velocity of 33 feet per second. The situation is modeled by the equation  representing the height in feet and t representing the time in seconds. Using this equation, define the domain of the ball when it reaches its maximum height. | 1.03 seconds |
| Q 2 | While hiking, Marek throws a rock off a cliff that is roughly 10 meters above a lake. This situation is modeled by the equation , with h representing the height in meters and t the time in seconds. If you were to set up a table to determine when the rock hits the water, what would be a reasonable scale for the range? | 2 |
| Q 3 | A high diver jumps off a platform 40 feet above the water. The height, h, in feet of the diver above the water at any time, t, in seconds is given by the function . Plot points to graph the equation. Which ordered pair represents the vertex? | (-2, 44); (1,56) |
| Q 4 | Luca is building a garden in his backyard. He needs to erect a fence to keep his dog Luna from digging in it. He has enough materials for a fence with 120 feet around the perimeter. This function is represented by the formula , where A is the area and w is the width. Which of the following correctly identifies the x-intercept(s)? | (0,0) and (60,0) |
| Q 5 | An owl swoops down from its perch in a tree at 30 feet to catch field mice. It follows the function , where t is the time in seconds and h is the height in feet. Which of the following **best** interprets the vertex? | (2, 6) |