Algebra 1

**1-Variable Equations & Inequalities**

**Unit Summary:** In this unit, you will learn to write and solve equations and inequalities in one-variable using properties of equality, as well as graphing methods. You will recall your knowledge of simplifying polynomial expressions.

**GeoGebra Math Practice Tool:** Math Practice is a tool for mastering algebraic notation. It supports students in their step-by-step math work, let's them explore different solution paths, and helps build confidence, fluency, and understanding.[*Teacher Guide*](https://help.geogebra.org/hc/en-us/articles/15294353125533-Teachers-Using-GeoGebra-Math-Practice-in-class) *|* [*Student Guide*](https://help.geogebra.org/hc/en-us/articles/15294377044381-Students-Learn-with-GeoGebra-Math-Practice) *|* [*Video Demo*](https://youtu.be/Injz3kiRx8g)

**Lesson 2 – True or False Statements**

**Key Words:**

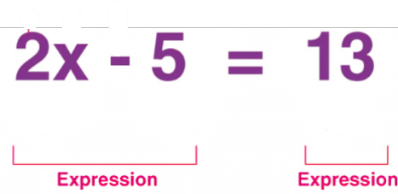
* **equation** – a usually formal statement of equality or equivalence in a mathematical expression
* **expression** – numbers, symbols and operators grouped together that show the value of something
* **inequality** – a statement of inequality between two quantities usually separated by a sign of inequality, such as < (is less than); > (is greater than); or ≠ (is not equal to)
* **simplify** – to perform all math operations possible until no more can be completed
* **statement** – a mathematical sentence that is either true or false
* **variable** – a quantity that may assume any one of a set of values, typically represented by a letter

**Objective 1:** In this section, you willdetermine the truth value of equations and inequalities.

*Mathematical Practice Standard: Construct viable arguments and critique the reasoning of others.*

**Big Ideas**:

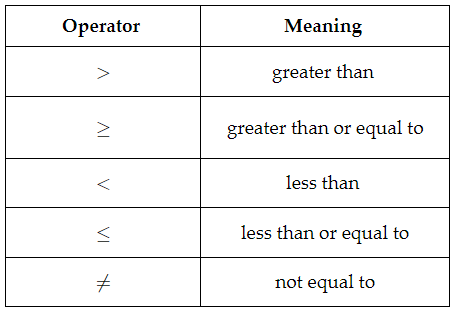
* Recall that an *equation* is made up of two *expressions* joined with an equal sign, also called a mathematical *statement*.



* In algebra, you will need to determine whether a given *equation* is true or false.
* To determine the truth value of an *equation*:
  + 1. Find the value of the expression on the **left side** of the equation.
  + 2. Find the value of the expression on the **right side** of the equation.
    - If the two values are **equal**, the equation is **true**.
    - If the two values are **not equal**, the equation is **false**.

|  |  |
| --- | --- |
| **True Equation** | **False Equation** |
| Determine the truth value of the equation: | Determine the truth value of the equation: |
| **1. Find the value of the left side:** | **1. Find the value of the left side:** |
| **2. Find the value of the right side:** | **2. Find the value of the right side:** |
| **3. Determine if true or false:**  The value of the left expression is equal to the value of the right expression, both are equal to 3, so the equation is true. | **3. Determine if true or false:**  The value of the left side of the equation is 10.5, and the value of the right side of the equation is 11. The equation is false. |

* Two *expressions* can also be compared in an *inequality*, which is a mathematical statement that uses the following symbols.



* For *inequalities*, you must still determine the value of each *expression* in the equation (the left side and the right side).
* Then you must use the *inequality* symbol to determine if the mathematical statement is true or false.

|  |  |
| --- | --- |
| **True Inequality** | **False Inequality** |
| Determine the truth value of the inequality: | Determine the truth value of the inequality: |
| **1. Find the value of the left side:** | **1. Find the value of the left side:** |
| **2. Find the value of the right side:** | **2. Find the value of the right side:** |
| **3. Rewrite the inequality:**  The mathematical statement reads: “13 is greater than or equal to 11.25.” | **3. Rewrite the inequality:**  The mathematical statement reads: “20 is less than 11.” |
| **4. Determine if true or false:**  13 is greater than or equal to 11.25, so the inequality is true. | **4. Determine if true or false:**  20 is not less than 11, so the inequality is false. |

**Objective 2:** In this section, you will determine whether the given variable value(s) make equations and inequalities true or false.

*Mathematical Practice Standard: Construct viable arguments and critique the reasoning of others.*

**Big Ideas:**

* Often, *equations* or *inequalities* will have a variable where you need to determine the value of the *variable* that makes a statement or *equation* true.
* To determine whether a given value or set of values makes an *equation* or *inequality* true or false:
  + 1. Substitute the given value or values into the *equation* or *inequality*, where the *variable* is.
  + 2. Find the values of the left and right sides of the *equation* or *inequality*.
  + 3. Determine whether the statement is true or false.

|  |  |
| --- | --- |
| **Example:** Which of these values will make the following equation true: 15, 20, or 25? | |
| **Step 1:** Substitute 15 for *x*. Determine if the value makes the statement true or false. | The values are not equal, the statement is **false**. |
| **Step 2:** Substitute 20 for *x*. Determine if the value makes the statement true or false. | The values are equal, the statement is **true**. |
| **Step 3:** Substitute 25 for *x*. Determine if the value makes the statement true or false. | The values are not equal, the statement is **false**. |
| **Step 4:** State the answer. | The only value for *x* that makes the equation true is 15. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Is the equation  true or false? Enter 1 if the statement is true. Enter 2 if the statement is false. | 1 |
| P 2 | Consider the following expressions:  Which operator (>, <, or =) should be inserted into the blank space to make the statement **true**?​ | > |
| P 3 | Consider this inequality: . Does the value of 3.5 for b make the inequality a **true** statement?  Enter 1 if the answer is yes. Enter 2 if the answer is no. | 1 |
| P 4 | Which value of *x* makes the equation  **true**: 20, 25, or 30?  x=\_\_\_\_\_ | 25 |
| P 5 | Consider the equation . Which of the following values for *z* makes the equation **true**: 11.5, 12.5, or 13.5?  The value of *z* that makes the equation true is \_\_\_\_\_. | 13.5 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Determine which of the following statements is/are **true**:  #1:  #2:  #3: | 1 and 3 |
| Q 2 | Which expression when substituted for *A* makes the equation ? |  |
| Q 3 | Consider the following expressions:  .  Which operator can be inserted to make the statement **true**? | > |
| Q 4 | When the value of *r* is 23, the equation  is a true statement. Which of the following values of *r* makes the inequality  a **true** statement? | 23.5 |
| Q 5 | Which of the following values for x makes the inequality  **true**? | 8 |

**Lesson 3 – Solution Sets of Equations & Inequalities**

**Key Words:**

* **compound inequality** – two inequalities joined by the word “and” or the word “or”
* **empty set** – a set that does not contain any elements
* **equation** – a usually formal statement of the equality or equivalence of mathematical or logical expressions
* **inequality** – a statement of inequality between two quantities usually separated by a sign of inequality, such as < (is less than); > (is greater than); or ≠ (is not equal to)
* **set notation**– the designation of a set, using braces {} to identify the solution or solutions to equations and inequalities
* **solution set** – the set of values that satisfy an equation
* **variable** – a letter used to represent an unknown quantity

**Objective 1:** In this section, you will use set notation to express the value(s) that make equations true.

*Mathematical Practice Standard: Attend to precision.*

**Big Ideas**:

* *Equations* that contain *variables* can have **one solution**, **many solutions**, or **no solutions**.
* You will need to [recall previous knowledge](#Bookmark1) of determining whether *equations* are true or false.
* The values that provide solutions to *equations* are known as the *solution set*.
  + A solution to an *equation* is any value that, when substituted for the *variable*, makes the *equation* true.
  + A *solution set* contains all the value(s) of the *variable* that makes the *equation* true.
  + The values of the *solution set* are included within a set of braces: **{}**
  + *Variables* are not included in the *solution set*, only the values that make the *equation* true.
    - For example, instead of {x=5}, the *set notation* is {5}.
* Review the appropriate *set notations* to use depending on the number of solutions:
  + **One solution** – only one value makes the statement true.
    - ex: , the solution set is **{1}**
  + **More than one solution** – more than one value makes the statement true.
    - ex: , the solution set is **{-1,1}**.
  + **Infinite Solutions** - any real number makes the statement true
    - ex: , the solution set is
  + **No solution** – no values make the statement true
    - ex: , the solution set is an **empty set {}**.

|  |  |
| --- | --- |
| **Example:** Show that 3 is the value of *x* that makes the equation true and express this using set notation. | |
| **Step 1:** Substitute 3 for *x a*nd simplify. |  |
| **Step 2:** Determine if 3 makes the equation true. | The value 3 for *x* makes the equation true, both sides have a value of 8. |
| **Step 3:** Express the solution in set notation. | The solution set is {3}. |

**Objective 2:** In this section, you will use set notation to express the value(s) that make inequalities true.

*Mathematical Practice Standard: Attend to precision.*

**Big Ideas:**

* Recall that the solutions to an *inequality* are the values that make the *inequality* true.
* Most *inequalities* have more than one value that will make them true and can be expressed using *set builder notation*.
* *Set builder notation* is used to describe the numbers that comprise the solution set and always places the following within braces:
  + 1. the relevant variable
  + 2. a colon symbol or a vertical bar to mean “such that”
  + 3. an inequality statement of the range of values of the variable that make the inequality true

|  |  |
| --- | --- |
| **Example** | **Set Builder Notation** |
| The solution is every number less than 5, but not including 5. | {: }  The set of all *x* such that *x* is less than 5. |
| The solution is every number greater than 5, but not including 5. | {: }  The set of all *x* such that *x* is gr  eater than 5. |
| The solution is every number less than 5 and including 5. | {: }  The set of all *x* is such that *x* is less than or equal to 5. |
| The solution is every number greater than 5 and including 5. | {: }  The set of all *x* such that *x* is greater than or equal to 5. |
| If an inequality has no solutions, the set notation would be {} which is called an empty set. | |

* A *compound inequality* contains two *inequalities* joined by the word “and” or the word “or.”

|  |  |
| --- | --- |
| **Example** | **Set Builder Notation** |
| Any number greater than or equal to 4 **or** less than or equal to –4 will make the inequality true. | {: }  The set of all *x* is such that *x* is less than or equal to –4 **or** greater than or equal to 4. |
| All values of x between –3 **and** 3 are solutions to this inequality. | {: }  The set of all *x* such that *x* is between -3 **and** 3. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Use set notation to express the value(s) that make the equation  true. Enter your response using set notation. | {10} |
| P 2 | Both −6 and 6 make the equation  true. Use set notation to express these solutions. When entering your response, list elements in the set from least to greatest, with a comma between each element in the set. | {-6, 6} |
| P 3 | The solution to the inequality  is . Use set notation to express the values of *y* that make the inequality  true.  {\_\_\_\_:\_\_\_\_} | y; y>4 |
| P 4 | Sophie, Adola, and Gilly are debating potential values in the solution set of the compound inequality . Sophie says that −2 is a solution. Adola says that 5 is a solution. Gilly says that 2 is a solution. Which person is correct? Enter 1 for Sophie, 2 for Adola, or 3 for Gilly.  The person who is correct is person \_\_\_\_\_. | 3 |
| P 5 | Which option correctly describes the values that make the inequality  true when written in set notation?  Option #1: {m:m<7}  Option #2: {7:7<m}  Option #3: {m<7:m}  Option #4: {m<7}  Option #\_\_\_\_ correctly states the values that make the inequality true in set notation. | 1 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Which set notation correctly expresses the solution set to the equation ? | {3} |
| Q 2 | Trey, Amelia, and Cho are discussing the solution set to the equation .  Trey says the solution is {6}.  Amelia says the solution is {−6,6}.  Cho says the solution is {−6}.  Which person has found the correct solution set? | Amelia |
| Q 3 | The values −1 and 1 make the equation  true. Use set notation to express the solution to the equation. | {-1, 1} |
| Q 4 | The solution set of the inequality  is the set of values of b greater than three. Which set notation correctly expresses the solutions? |  |
| Q 5 | What value falls in the solution set of the inequality ? | -1 |

**Lesson 4 – Solving Linear Equations**

**Key Words:**

* **Addition Property of Equality** – a rule stating that adding the same number to both sides of an equation produces an equivalent equation
* **Division Property of Equality** – a rule stating that dividing both sides of an equation by the same nonzero number produces an equivalent equation
* **equation** – a usually formal statement of the equality or equivalence of mathematical or logical expressions
* **identity** – a state of equality for all values of the variables between the left-hand side and the right-hand side of an equation
* **inverse operations** – a pair of mathematical operations, such as addition and subtraction, that undo one another
* **linear equation** – an equation containing a variable raised to the power of 1; an equation in the form ax+b=0 where a≠0
* **Multiplication Property of Equality** – a rule stating that multiplying both sides of an equation by the same number produces an equivalent equation
* **solution** – a set of values of the variables that satisfies an equation; for an equation with one variable, a value (number) that, when substituted in for the variable, makes a true statement
* **solution set** – the set of values that satisfy an equation
* **Subtraction Property of Equality** – a rule stating that subtracting the same number from both sides of an equation produces an equivalent equation
* **variable** – a quantity that may assume any one of a set of values, typically represented by a letter

**Formulas:**

* Addition Property of Equality: If , then
* Subtraction Property of Equality: If , then
* Multiplication Property of Equality: If , then
* Division Property of Equality: If , then

**Objective 1:** In this section, you will use properties of equality to justify each step in the process of solving linear equations.

*Mathematical Practice Standard: Construct viable arguments and critique the reasoning of others.*

**Big Ideas**:

* [Recall](#Bookmark2) that, when solving equations, some have one solution, no solution, many solutions, or infinite solutions.
* Sometimes, an *equation* will have multiple operators such as multiplication, addition, subtraction, and/or division.
* *Properties of equality* allow you to perform operations on both sides of an equation to create equivalent equations.
* When balancing equations, you will use the opposite operation, known as *inverse operations*.
* Addition and subtraction are *inverse operations*.
* Division and multiplication are *inverse operations*.

|  |
| --- |
| **Addition Property of Equality** |
| A rule stating that adding the same number to both sides of an equation produces an equivalent equation or inequality.  If , then  Use when the operator in the equation is **subtraction**. |
| **Example**: Solve for x using properties of equality.  The operator in this equation is subtraction. **Subtractions inverse operation is addition**, so we will use the ***Addition Property of Equality*** to balance the equation and solve for *x*.  To determine the value of *x*, add 5 to both sides of the equation. |

|  |
| --- |
| **Subtraction Property of Equality** |
| A rule stating that subtracting the same number to both sides of an equation produces an equivalent equation or inequality.  If , then  Use when the operator in the equation is **addition**. |
| **Example:** Solve for x using properties of equality.  The operator in this equation is addition. **Additions inverse operation is subtraction**, so we will use the ***Subtraction Property of Equality*** to balance the equation and solve for *x*.  To determine the value of *x*, subtract 10 from both sides of the equation. |

|  |
| --- |
| **Multiplication Property of Equality** |
| A rule stating that multiplying both sides of an equation by the same number produces an equivalent equation or inequality.  If , then  Use when the operator in the equation is **division**. |
| **Example:** Solve for x using properties of equality.    The operator in this equation is division. **Divisions inverse operation is multiplication**, so we will use the ***Multiplication Property of Equality*** to balance the equation and solve for *x*.  To determine the value of x, multiply both sides of the equation by 12. |

|  |
| --- |
| **Division Property of Equality** |
| A rule stating that dividing both sides of an equation by the same nonzero number produces an equation or inequality.  If , then  Use when the operator in the equation is **multiplication**. |
| **Example:** Solve for x using properties of equality.  The operator in this equation is multiplication. **Multiplications inverse is division**, so we will use the ***Division Property of Equality*** to balance the equation and solve for *x*.  To determine the value of x, divide both sides of the equation by 25. |

* You will encounter *equations* that require more than one property of equality to solve.
* Let’s use an example to that uses many properties of equality to solve:

|  |  |
| --- | --- |
| **Example:** What value of x will make the following equation true? | |
| **Step 1:** Use the Distributive Property on expressions in parentheses where necessary. | Distribute the 2 among the expression (). |
| **Step 2:** Simplify each side of the equation by combining like terms. | The right side is already simplified. Combine like terms on the left side of the equation. |
| **Step 3:** Work to move all terms with the variable x to the left side of the equation.Use the Subtraction Property of Equality. | Use the Subtraction Property of Equality on +2x to see what happens. |
| **Step 4:** Work to isolate the term with the variable *x*. Use the Addition Property of Equality. | Use the Addition Property of Equality on –14 to see what happens. |
| **Step 5:** Solve for *x*. Use the Division Property of Equality. | 6 and x are attached by multiplication. To separate them, we use the inverse operation division.  Use the Division Property of Equality on 6 to see what happens.    is equivalent to 1, so the left side becomes x.  , so the right side becomes 3. |

**Objective 2:** In this section, you will identify linear equations that have the same solution set.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* [Recall](#Bookmark3) how to use properties of equality to solve *equations*.
* Sometimes a pair of *linear equations* will appear to be different but have the same *solution set*.
  + The *equations* may have different operations and variables but the same *solutions*.
  + When different *equations* have the same *solution set*, the same values of the *variable* make both *equations* true statements.
  + To determine if *equations* have the same *solution set* you need to solve for the *variable* in each *equation*.

|  |  |
| --- | --- |
| **Example:** Determine which equations have the same solution set. | |
| **Step 1:** Use properties of equality to solve for *x* in the *first* equation. | The solution set is {3} |
| **Step 2:** Use properties of equality to solve for *x* in the *second* equation. | The solution set is {5}. |
| **Step 3:** Use properties of equality to solve for *x* in the *third* equation. | Subtraction Property of Equality    Division Property of Equality    The solution set is {5}. |
| **Step 4:** State the answer. | The solution set for the first equation is {3}, while the solution set for the second and third equations is {5}. |

* Substitution can also be used to check if the *solution* of one *equation* is the *solution* for a different *equation*.
  + You can solve one *equation* at a time and substitute the values in its *solution set* into the other *equations*.

|  |  |
| --- | --- |
| **Example:** Determine whether the following equations have the same solution set. | |
| **Step 1:** Use the properties of equality to find the solution set of the first equation. | The solution set is {8}. |
| **Step 2:** Use substitution to check if the solution set of the **first** equation is true for the **second** equation. | The solution set of the first equation is {8}.  Substitute 8 for *x* and simplify each side of the equation.    73 is not equal to 7, meaning the solution set {8} is not the solution set for this equation. |
| **Step 3:** Use substitution to check if the solution set of the **first** equation is true for the **third** equation. | The solution set of the first equation is {8}.  Substitute 8 for *a* and simplify each side of the equation.    48 is equivalent to 48, this is a true statement. Therefore, the solution set {8} is also a solution for the third equation. |
| **Step 4:** State the answer. | The first and third equations have the same solution set of {8}. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | What value will make the equation true? Write your answer as a fraction.  The value for *m* that makes the equation true is m=\_\_\_\_\_. |  |
| P 2 | Using the properties of equality, solve the equation .  The solution to the equation is *x*=\_\_\_\_\_. | 2 |
| P 3 | Sehyr solved the equation . She seems to have made an error in her work because the solution she found does not work. Observe Sehyr’s work to find her error and correct it to find the solution to the equation.  Sehyr’s work:  Using the Addition Property of Equality  Using the Division Property of Equality  The solution to the equation is \_\_\_\_\_. | 36 |
| P 4 | How many of the following equations have the solution set {5}?  The number of equations that have the solution set {5} is \_\_\_\_. | 2 |
| P 5 | What value is a solution to two of the equations?  The solution set {\_\_\_\_\_} is the solution to two equations. | 12 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Which property of equality justifies the step for solving the equation that is displayed? | Addition Property of Equality |
| Q 2 | Using the properties of equality, find the solution to the equation . | 4 |
| Q 3 | How many solutions will the equation  have? | infinite/many solutions |
| Q 4 | Identify the pair of linear equations that have the same solution set. |  |
| Q 5 | Tamara, Payton, Krish, and Haruki are analyzing the solution sets to the following linear equations. Tamara says that Equation A and Equation C have the same solution. Payton says Equation B and Equation C have the same solution. Krish says all three have the same solution. Haruki says that none of the equations have the same solution. Identify who is correct about the linear equations.  Equation A:  Equation B:  Equation C: | Haruki |

**Lesson 5 – Solving Linear Inequalities**

**Key Words:**

* **Addition Property of Inequality** – a rule stating that adding the same number to both sides of an inequality produces an equivalent inequality
* **Division Property of Inequality** – a rule stating that dividing both sides of an inequality by the same positive number produces an equivalent inequality
* **inequality** – a statement of inequality between two quantities usually separated by a sign of inequality, such as < (is less than); > (is greater than); or ≠ (is not equal to)
* **linear inequality** – an expression in which two linear expressions are compared using the inequality symbols (>, <, ≥, ≤, ≠)
* **Multiplication Property of Inequality** – a rule stating that multiplying both sides of an equality by the same positive number produces an equivalent inequality
* **Subtraction Property of Inequality** – a rule stating that subtracting the same number from both sides of an inequality produces an equivalent inequality

**Formulas:**

* Addition Property of Inequality: If , then
* Subtraction Property of Inequality: If , then
* Multiplication Property of Inequality:
  + If and , then
  + If and , then
* Division Property of Inequality:
  + If and and , then
  + If and and , then

**Objective 1:** In this section, use a process of justifying each step with the properties of inequality to solve linear inequalities.

*Mathematical Practice Standard: Construct viable arguments and critique the reasoning of others.*

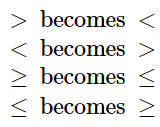
**Big Ideas**:

* You have previously used properties of equality to solve linear equations. These properties also apply to solving *linear inequalities*.

|  |
| --- |
| **Addition Property of Inequality** |
| A rule stating that adding the same number to both sides of an equation produces an equivalent equation or inequality.  If , then |

|  |
| --- |
| **Subtraction Property of Equality** |
| A rule stating that subtracting the same number to both sides of an equation produces an equivalent equation or inequality.  If , then |

* It’s important to note the *Multiplication and Division Properties of Inequality* only hold true when multiplying or dividing by positive numbers.
* If you do multiply or divide both sides of an inequality by a negative number, then you will need to **flip the inequality symbol** to create an equivalent *inequality*.



|  |
| --- |
| **Multiplication Property of Equality** |
| A rule stating that multiplying both sides of an equation by the same number produces an equivalent equation or inequality.  If and , then  If and , then  \*If you multiply or divide both sides of an inequality by a negative number, the symbol flips. |

|  |
| --- |
| **Division Property of Equality** |
| A rule stating that dividing both sides of an equation by the same nonzero number produces an equation or inequality.  If and , then  If and , then  \*If you multiply or divide both sides of an inequality by a negative number, the symbol flips. |

* Recall that several properties can be used to solve an equation or inequality.

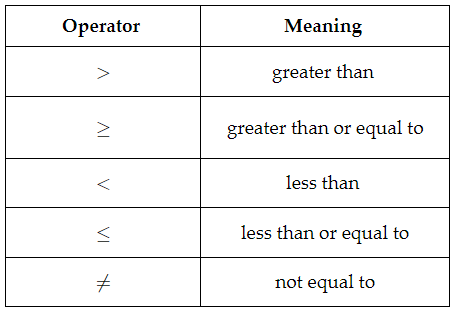
|  |  |
| --- | --- |
| **Example**: Solve the inequality. | |
| **Step 1**: Use the Subtraction Property of Inequality. |  |
| **Step 2**: Use the Addition Property of Inequality. |  |
| **Step 3**: Use the Division Property of Inequality. |  |
| **Step 4**: Express the values that make the statement true in set notation. | The solution set is . |

**Objective 2:** In this section, you will graph solution sets for linear inequalities on a number line.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* Recall that an *inequality* is a formal statement of *inequality* between two quantities that are separated by a sign:



* The solution sets of *inequalities* can be graphed on a number line using the following rules.
  + Shaded/solid line: indicate values that are included in the solution set
  + Unshaded values: indicate values that are not included in the solution set
  + Closed circle: indicates the value is included in the solution set
  + Open Circle: indicates value is not in the solution set
* Follow these steps to graph the solution sets of *inequalities*:
  + 1. Examine the *inequality* symbol to determine if the graph is an open interval (open circle) or closed interval (closed circle).
  + 2. Place the circle on the graph at the given number from the *inequality*.
  + 3. Shade the solution set. Think about what values make this *inequality* true.

|  |  |
| --- | --- |
| **Inequality** | **Graph of Solution** |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Solve the inequality .  The solution to the inequality is *d*≤\_\_\_\_\_. | 30 |
| P 2 | Use the properties of inequality to find the solution to .  The solution to the inequality is 𝑘≥\_\_\_\_. | 3 |
| P 3 | Use the image to answer the question.  What inequality is depicted in the graph?  *x* \_\_\_\_\_ |  |
| P 4 | Use the image to answer the question.  What inequality is depicted in the graph?  *x* \_\_\_\_\_ |  |
| P 5 | Use the image to answer the question.  What inequality is depicted in the graph?  *x* \_\_\_\_\_ |  |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | What property of inequality justifies the steps shown in the following solution process? | Multiplication Property of Inequality |
| Q 2 | What property of inequality can justify the math in Step 2?  Step 1:  Step 2:  Step 3: | Subtraction Property of Inequality |
| Q 3 | Juana, Anish, and Giselle are comparing their solutions to the inequality . Juana says that the inequality symbol must be reversed because there is a negative sign in the inequality. Anish says the Multiplication Property of Inequality or Division Property of Inequality must be applied. Giselle says the inequality symbol must stay the same.  Based on these answers, which combination of people is correct? | Giselle and Anish |
| Q 4 | Use the image to answer the question.  What inequality does the number line graph? |  |
| Q 5 | Graph . |  |

**Lesson 6 – Multiple Equations or Inequalities**

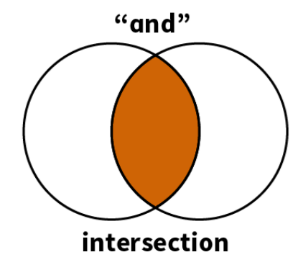
**Key Words:**

* **compound inequality** – a pair of two or more inequalities joined with the words “and” or “or”
* **inequality** – a statement of inequality between two quantities usually separated by a sign of inequality, such as < (is less than); > (is greater than); or ≠ (is not equal to)
* **intersection of two sets** – the set of elements common to both sets
* **union of two sets** – the set of elements that are in either or both of the original sets

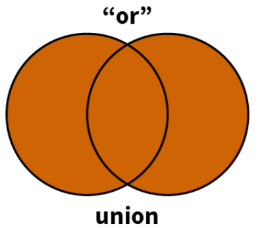
**Objective 1:** In this section, you will determine solution sets for two or more equations or inequalities joined by "and" or "or."

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas**:

* *Compound inequalities* are a pair of two or more *inequalities* joined with the words “**and**” or “**or**”.
* If two equations or *inequalities* are joined with the word “and,” the solution set is the elements that the two sets have in common.
  + The word “and” means the same as the ***intersection of two sets.***
  + 
* The steps to solve an equation or *inequality* joined with an “**and**” operator are:

|  |  |
| --- | --- |
| **Example:** Find the solution set. | |
| **Step 1:** Separate the two equations or inequalities. | Inequality #1:  Inequality #2:  Inequality #3: |
| **Step 2:** Find the solution set of each individual equation or inequality. | |  |  |  | | --- | --- | --- | | Inequality #1 | Inequality #2 | Inequality #3 | | The solution set is | The solution set is | The solution set is | |
| **Step 3:** Determine the elements of the solution sets that are **common to all sets**. Which values make all three equations true? | The three solution sets are:  The elements of the solution sets that are common to all sets are numbers that are both greater than or equal to –1 and less than or equal to 2.  The solution set is: |

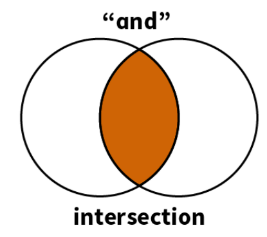
* The steps to solve an equation or *inequality* joined with an “**or**” operator are:
  + 1. Separate the two equations or inequalities.
  + 2. Find the solution set of each individual equation or inequality.
  + 3. Determine the elements of the solution sets that are common to both sets. Which values make both equations true?
* If two equations or *inequalities* are joined with the word “**or**,” the solution set is the elements that exist in any of the individual solution sets or the *union* of the *solution sets*.
  + 
* The steps to solve an equation or *inequality* joined with an “**or**” operator are:

|  |  |
| --- | --- |
| **Example:** Find the solution set.  -4 | |
| **Step 1:** Separate the two equations or inequalities. | Equation:  Inequality: -4 |
| **Step 2:** Find the solution set of each individual equation or inequality. | |  |  | | --- | --- | | Equation | Inequality | | The solution set is {-3}. | The solution set is {*y: y>*-3}. | |
| **Step 3:** Determine the set of elements that exist in either or both individual solution sets. | The union of the two sets, or the elements that exist in either of the individual solution sets include –3.  The solution set is: |

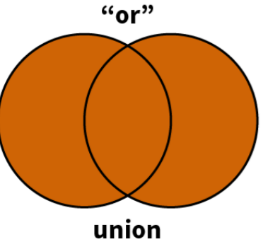
**Objective 2:** In this section, you will graph solution sets on a number line for two or more equations or inequalities joined by *and* or *or*.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas:**

* Recall that the “**and**” operator represents the *intersection of two sets*, or elements common to both sets.
  + 
* To graph the solution sets of *inequalities* joined by “**and**” use the following steps:
  + Sometimes, your first step will be to solve each equation or inequality for the variable.

|  |  |
| --- | --- |
| **Example:** Graph the following solution set on a number line. | |
| **Step 1:** Graph the inequality on a separate number line. |  |
| **Step 2:** Graph the inequality on a separate number line. |  |
| **Step 3:** Determine the elements common to both graphs. | All numbers between –2 and –1, including –2 and –1, are common to both graphs.    The solution set is: |

* Recall that the “**or**” operator represents the *union of two sets*, or elements that exist in any of the original sets.
  + 
* To graph the solution sets of *inequalities* joined by “**or**” use the following steps:
  + Sometimes, your first step will be to solve each equation or inequality for the variable.

|  |  |
| --- | --- |
| **Example:** Graph the solution set on a number line. | |
| **Step 1:** Solve the equation and graph the solution on a number line. |  |
| **Step 2:** Solve the equation and graph the solution on a number line. |  |
| **Step 3:** Determine the set of the elements that exist in any of the original sets and graph the solution on a number line. | The set of elements that exist in any of the original sets is all numbers greater than 4 or less than or equal to –3. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | What is the solution set of  and ?  {𝑥|𝑥\_\_\_\_} | ≥4 |
| P 2 | What is the solution set of  and ?  {\_\_\_} | 4 |
| P 3 | Use the image to answer the question.  What is the solution set to the inequalities on the graph?  𝑥<\_\_\_\_ or 𝑥>\_\_\_\_ | 0; 2 |
| P 4 | Use the image to answer the question.  To indicate the solution for  and , where would you place circles on the graph? Enter the lesser value first. | -1; 4 |
| P 5 | Use the image to answer the question.  What solution set does the graph represent?  𝑥 ≤ \_\_\_\_ and 𝑥≥ \_\_\_ | 7; -2 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Determine the solution set of  and −. | {-1} |
| Q 2 | Determine the solution set of  or . | {-1, 3} |
| Q 3 | Use the image to answer the question.  What solution set does the number line graph? | or |
| Q 4 | Use the image to answer the question.  Select the solution set represented on the graph. | and |
| Q 5 | Use the image to answer the question.  Select the solution set represented on the graph. | or |

**Lesson 7 – Variable Expressions in Denominators**

**Key Words:**

* **undefined** – an expression that has no meaning
* **variable** – a quantity that may assume any one of a set of values, typically represented by a letter

**Objective 1:** In this section, you will rewrite equations containing variables in denominators as two equations joined by "and."

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas**:

* Any fraction with a denominator equal to zero is *undefined or* has no meaning.
  + For example: .
* When an equation contains a *variable* in a denominator, the fraction will become *undefined* if the denominator equals zero.
* To prevent this from happening, rewrite the equation by joining with an “and,” along with the variable the value that would cause the denominator to equal zero.
  + The first equation is the same as the original.
  + The second equation specifies the values that would cause a fraction within the equation to be *undefined*.

|  |  |
| --- | --- |
| **Variables in the Denominator** | |
| Example:Rewrite the equation as two equations joined by “and.” | |
| **Step 1:** Think about what the equation is telling you. | * There is some value of *x* that will make the equation true. * The value of *x* cannot make the fraction *undefined*. * The fraction is undefined if , is equal to zero, so . |
| **Step 2:** Determine the *x* value that makes the fraction undefined. |  |
| **Step 3**: Write as two equations joined by “and”. |  |

**Objective 2:** In this section, you will solve equations that include a variable in the denominator.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* Recall that to find the solution set to any equation, you must determine the values of the *variable* that make the equation true.
* [Recall](#Bookmark4) how to rewrite equations as two equations joined by “and” in previous example.
* Now, using the same example with a *variable* in the denominator, we will solve it.

|  |  |
| --- | --- |
| [Example](#Bookmark5): Solve . | |
| **Step 1:** Rewrite as two equations, done in the previous example. |  |
| **Step 2:** Solve the first equation using the properties of equality. | The value 0 makes the equation, , true. |
| **Step 3:** Consider the second equation. | Does the value 0 also make this equation true?  Yes, because . |
| **Step 4:** State the solution set. | The solution set for is {0}. |

* Remember that the *denominator* of a fraction cannot be equal to zero, but the *solution set* can include zero if it makes both equations true.
* Sometimes, solutions that are found when solving the first equation do not end up in the final solution set.
  + Any values that cause a fraction within the equation to be *undefined* cannot be in the solution set of the equation.

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Rewrite the equation  as two equations joined by “and.”   and *b*≠ \_\_\_\_\_ | 0 |
| P 2 | What value will complete the set of equations  and 𝑥≠\_\_\_\_\_?  The value that completes the set of equations is 𝑥≠ \_\_\_\_\_. | 3 |
| P 3 | What is the undefined value for ?  Write your answer in reduced fraction form, if applicable.  The undefined value is \_\_\_\_. |  |
| P 4 | Solve for x in the following equation: Write your answer in reduced fraction form, if applicable.  𝑥= \_\_\_\_ |  |
| P 5 | What is the value of the variable p in the given equation?  Write your answer in reduced fraction form, if applicable.  𝑝= \_\_\_ |  |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Rewrite the equation  as two equations joined by “and.” |  |
| Q 2 | What happens when a value causes a denominator in a fraction to be equal to zero? | This fraction is undefined. |
| Q 3 | What value makes the equation  undefined? |  |
| Q 4 | Solve the equation for the variable k: | 10 |
| Q 5 | What is the value for the variable*f*in ? | 6 |

**Lesson 8 – Rearranging Formulas**

**Key Words:**

* **inverse operations** – a pair of mathematical operations, such as addition and subtraction, that undo one another
* **quantity of interest** – the variable that the equation is equal to
* **term** – is either a single number or variable, or numbers and variables multiplied together
* **unit** – a determinate quantity (as of length, time, heat, or value) adopted as a standard of measurement

**Objective 1:** In this section, you will use the properties of equality to rearrange formulas to highlight different quantities of interest.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas**:

* [Recall the properties of equality used to solve equations.](#Bookmark3)
* These properties can also be used to rearrange a given formula to highlight a specified *quantity of interest*.
  + A *quantity of interest* means your equation will be equal to that *variable*.
  + For example, consider the formula for the area of a rectangle, . This formula can be rewritten to highlight different quantities of interest.

|  |
| --- |
| Rearrange the formula to highlight length, , as the *quantity of interest*.    This is equivalent to .  The two formulas and . |

* You can apply this process to any formula if you need to isolate, or highlight, a specific *variable*.
  + The different versions of the formula are equivalent, and each highlights a different *quantity of interest*.
* Let’s consider one more example:

|  |  |
| --- | --- |
| **Example:** Rearrange the formula for the perimeter of a rectangle, , to highlight width, *w*. | |
| **Step 1:** Identify the quantity of interest. | The quantity of interest is *w*. Your goal is to get w along on one side of the equation. |
| **Step 2:** Use the properties of equality to rearrange the formula. | 1. Subtraction Property of Equality.      1. Division Property of Equality |
| **Step 2:** Write the equivalent formula. | This is equivalent to . This version of the formula gives you width in terms of perimeter and length. |

**Objective 2:** In this section, you will use units to help justify that rearrangements of formulas make sense.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas:**

* You can use *units* to justify that a rearrangement of a formula makes sense.
  + Recall that *units* define a quantity such as length, time, heat, or value.
* This is done by rearranging the formula to highlight a quantity of interest and using units to help justify the rearrangement.

|  |  |
| --- | --- |
| **Example:** The formula gives the distance an object travels, *d*, in terms of its velocity, *v*, and time spent traveling, *t*. | |
| **Step 1:** Rearrange this formula to highlight *v* as the quantity of interest. |  |
| **Step 2:** Identify the units that represent each variable. | The variables in this formula represent the following units:   * velocity, v, is measured in meters per second: * distance, d, is measured in meters: * time, t, is measured in seconds: |
| **Step 3:** Substitute the units for the variables in the formula. |  |
| **Step 4:** Confirm that the units make sense. | is a true statement and confirms that this rearrangement of the formula makes sense. |

* In some cases, you will need to convert units so that they match prior to using them in a formula.

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Rearrange the formula  using m as the quantity of interest. |  |
| P 2 | The surface area of a square pyramid can be calculated using the equation . Rearrange the surface area equation for the quantity of interest 𝐵.  𝐵= \_\_\_ |  |
| P 3 | Claudia’s family is buying a new cabinet for their home. They need to know the width of the cabinet to make sure that it will fit in the space where they want to put it. Using the volume formula, 𝑉=𝑙𝑤ℎ, rearrange the formula to highlight the quantity of interest. Note that volume is represented with a capital 𝑉 in the formula. |  |
| P 4 | Amelia is traveling from Los Angeles, California, to Mesa, Arizona. She is wondering at what speed she will have to travel in order to make that happen. Using the formula 𝑑=𝑣𝑡, rearrange the formula to highlight the quantity of interest. |  |
| P 5 | A team of zoologists collected data on the velocity of falcons using a speedometer. The speedometer measured that a falcon was flying at 68 mph. The zoologists are interested in finding out how many hours it would take for the falcon to fly 52,800 feet if it continues to fly at a constant speed of 68 mph. In order to use units to verify your rearrangement of the formula, what process needs to be done? Enter the value that corresponds to your answer.  Option #1: miles  Option #2: yards  Option #3: hours  I would need to convert the units of distance from feet to Option # \_\_\_. | 1 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Using the Property of Equality, what first step would you take to rearrange the formula  using x as the quantity of interest? | Subtract *b* from both sides of the equation. |
| Q 2 | Which of the following is an example of using the Division Property of Equality to rearrange the equation ? |  |
| Q 3 | Given the formula for the circumference of a circle, , how would you rearrange the formula if you were interested in finding the radius of the circle? |  |
| Q 4 | Given the formula for the volume of a box, , what operation would you use to isolate the width? | in. = in. division |
| Q 5 | Sammy wants to calculate the time it takes for a ladybug traveling at a speed of 3 inches (in.) per second (sec.) to crawl a distance of 10 feet (ft.).  Starting with the formula , Sammy rearranges the formula to highlight time, *t*, as the quantity of interest.  Sammy substitutes the values for rate and time into the formula and simplifies to calculate the time.  Sammy determines it takes the ladybug about 3.3 sec. to travel 10 ft.  What mistake did Sammy make? | Sammy needed to convert 10 ft. to 120 in. by multiplying by before substituting. |

**Lesson 9 – Creating One-Variable Equations & Inequalities**

**Key Words:**

* **constraint** – a restriction or limitation that is placed on variables used in equations that use real-world situations
* **equation** – a usually formal statement of the equality or equivalence of mathematical or logical expressions
* **inequality** – a statement of inequality between two quantities usually separated by a sign of inequality, such as < (is less than); >(is greater than); or ≠ (not equal to)
* **nonviable** – a result not possible as a solution to the problem with constraints
* **solution** – a set of values of the variables that satisfies an equation; for an equation with one variable, a value (number) that, when substituted in for the variable, makes a true statement
* **solve** – to find a solution, explanation, or answer for
* **variable** – a quantity that may assume any one of a set of values, typically represented by a letter
* **viable** – a result is possible as a solution to a problem with constraints

**Objective 1:** In this section, you will use equations created with one variable to solve problems.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas**:

* Whenever you come across a word problem with one unknown or *variable* there is a set of steps to follow.
* Many word problems require you to create *equations* in one variable that you then solve to find one answer.
* Other word problems require you to create statements that cover a range of possible answers using *inequalities*.
* These steps will help you create and use *one-variable* *equations or inequalities* to solve word problems.
  + Solving word problems using *equations* involves a series of steps.
    - 1. Determine what the problem is telling you.
    - 2. Determine what the problem is asking you to find.
    - 3. Create an *equation* to represent the problem, using a *variable* to represent the unknown value or what you need to find.
  + Once you have set up your equation, take the following steps to *solve* it.
    - 1. Develop a plan.
    - 2. Discover the values of the variable that make the statement true.
    - 3. Express the values that make the statement true in set notation.

|  |  |
| --- | --- |
| **Example:** Create an equation and solve the following problem.  Your school yearbook is selling half-page ads to seniors. There are 18 pages available in the yearbook, but 5 of those pages have been designated for clubs and team photos. Considering all the information provided, how many ads can be sold to seniors? | |
| **Step 1:** What is the problem telling you? | * 18 yearbook pages * 5 for club and team photos * senior ads will be page |
| **Step 2:** What is the problem asking you to do? | Determine how many senior ads can be sold for the yearbook. |
| **Step 3:** Create an equation. | * Let represent the number of senior ads that can be sold. will be the coefficient for since each ad is page. * You also need to add a constant of 5 since that is how many pages are already taken up. * The equation should be equal to 18 pages, the maximum number of available pages. |
| **Step 4:** Develop a plan and discover the values of the variable that make the statement true by solving the equation. |  |
| **Step 5:** Express the values that make the statement true in set notation. | The solution set is {26}, so you can conclude that 26 senior ads can be sold. |

**Objective 2:** In this section, you will represent constraints of a contextual situation by interpreting solutions of equations and inequalities as viable or nonviable.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* When working with real-world scenarios sometimes the solutions are impractical because the solution may not fit the context of the problem.
  + For example, if your problem involves running a race and asks for the average speed, a negative answer will not fit the context of the problem because you cannot run at a negative speed.
* A context that limits the solution set to an *equation* or an *inequality* is called a *constraint.* 
  + A solution that makes a true statement and fits the *constraints* of a problem is called a *viable solution*.
  + Solutions that do not fit those *constraints* are called *nonviable*.

|  |  |
| --- | --- |
| **Viable Solution** | |
| Example:Solve the given equation and determine whether the solution is viable or nonviable.  A chef is planning a meal and will serve each guest cup of rice. She has 5 cups of rice and must use all of it. How many people can she serve? | |
| **Step 1:** Create an equation that represents the scenario. | * Let *c* represent the number of servings. The coefficient is because each serving is cup. * The chef only has 5 cups of rice in total.   Equation: |
| **Step 2:** Solve the equation for the variable. | The solution shows there can be 15 servings before the chef runs out of rice. |
| **Step 3:** Determine if the solution is viable. | The chef will get 15 servings from the 5 cups of rice, so 15 people can be served using all of it. This is a viable solution. |

|  |  |
| --- | --- |
| **Non-Viable Solution** | |
| Example: Solve the given inequality and determine whether the solution is viable or nonviable.  A family needs to buy plane tickets. They go online where the tickets cost $187 per ticket plus an additional $19 per ticket for travel insurance, which the family will also purchase. The website charges a fee of $24 for the entire purchase. The family needs 5 tickets and has a total of $1,000. Will they be able to purchase all 5 tickets? | |
| **Step 1:** Create an equation that represents the scenario. | * Let *p* represent the number of tickets purchased. One term is 187*p* to represent the cost of the ticket. One term is 19*p* to represent the travel insurance cost per ticket. * The constant, 24, represents the flat fee for buying any amount of tickets. * The family can’t spend more than $1,000.   Inequality: |
| **Step 2:** Solve the inequality for the variable. | This solution tells us that the family can purchase up to 4.73 tickets to stay under their budget of $1,000. |
| **Step 3:** Determine if the solution is viable. | No, they cannot purchase 5 tickets. This solution is nonviable.  The family would only be able to purchase 4 tickets. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | A large bin can store up to 44 towels. A small bin can hold three-fourths the volume of a large bin. You have already put 17 in the small bin. How many more towels can you store?  You can store up to ­­\_\_\_ more towels. | 16 |
| P 2 | You are playing a new video game. It says that you have completed  of the game. You have played for 23 minutes. Write and solve an equation to find how long it will take to complete your video game at this rate.  It will take \_\_\_ minutes to complete the game. | 115 |
| P 3 | Oksana and her three friends are heading to a concert. They paid $540 in total for tickets. This included insurance for $15 for each person. How much was the cost of one ticket to the concert?  The cost for the one ticket to the concert was $ \_\_\_. | 120 |
| P 4 | Christiana makes $14 per hour working at the roller-skating rink. She needs to make $130 to pay her car insurance. How many hours should she work if she receives an $18 tip for hosting a birthday party that day?  She should work \_\_\_ hours to make $130. | 8 |
| P 5 | A large dehumidifier removes 1.6 pints of water from the air around it per hour. The dehumidifier can hold up to 40 pints of water. Which option number represents the inequality that shows how much time the dehumidifier can run before it becomes full?  Option #1:  Option #2:  Option #3:  Option #4:  Option # \_\_\_\_ represents how long the dehumidifier can run before it becomes full. | 2 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Alexei is stocking a shelf at the store. The shelf can hold 58 cans, so he can still put 39 cans on the shelf before it is full. Create an equation to make sense of the problem and solve it to find out how many cans are already on the shelf. Which equation and answer are correct? | and 19 |
| Q 2 | Trey is running in a race. He has completed  of the race so far. He has already run 1.2 miles. How many more miles must Trey run to complete the race? | 3.6 miles |
| Q 3 | Carlos is playing basketball this season. He is trying to average 25 points per game. He has scored 27, 18, 24, 32, 15, and 27 points in the previous 6 games. What equation can help you find the score for the last game that will give Carlos an average of 25 points? |  |
| Q 4 | The adventure club is selling candy bars to go on their annual outing. The bars sell for $2 a bar. The club bought 725 bars for $400. The club needs to make at least $1,000 to go on their trip. Which inequality best represents this problem, and are the 725 bars a viable option to make enough profit? | , and yes, it is a viable option. |
| Q 5 | A horse trainer has 42 horse treats. She needs five treats per horse. Which equation shows the number of horses, h, that she can train, and is the solution viable or nonviable? | , so horses, which is nonviable. |