Geometry

**Coordinate Geometry**

**Unit Summary:** In this unit, you will learn to use Pythagorean Theorem to calculate distance between two coordinates on a coordinate plane and midpoints. Using coordinates, you will learn to calculate the perimeter and area of shapes. Finally, you will learn how to calculate and prove geometric theorems algebraically using slope criteria of lines.

**Lesson 2 – Distance in the Coordinate Plane**

**Key Words:**

* **adjacent sides** – two sides of a polygon that share a common vertex
* **coordinates** – numbers that describe the position of points along certain dimensions
* **distance formula** – a formula that uses the coordinates of two points to determine the distance between them on the coordinate plane
* **line segment** – part of a line that has two endpoints
* **ordered pair** – the two coordinates that name the location of a point on the coordinate plane
* **parallelogram** – a four-sided polygon with opposite sides that are parallel and equal in length
* **perimeter** – the continuous line forming the boundary of an enclosed geometric figure
* **polygon** – a closed figure made up of line segments in a two-dimensional plane
* **Pythagorean Theorem** – the theorem which states that the square of the length of the hypotenuse of a right triangle is equal to the sum of the squared lengths of the other two sides
* **vertex** – a point (as of an angle, polygon, polyhedron, graph, or network) that terminates a line or curve or comprises the intersection of two or more lines or curves
* **x-coordinate** – the first coordinate in an ordered pair that tells the distance to travel left or right from the origin
* **y-coordinate** – the second coordinate in an ordered pair that tells the distance to travel up or down from the origin

**Formulas:**

* Pythagorean Theorem:
* Distance Formula:

**Objective 1:** In this section, you will use the Pythagorean Theorem to develop the formula for the distance between two points on the coordinate plane.

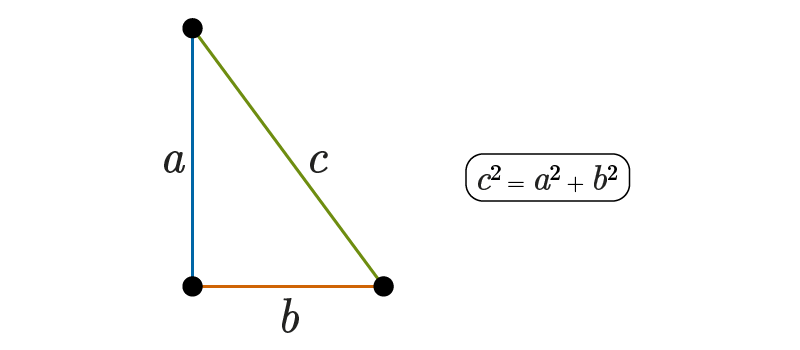
*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas**:

* The distance between two points is the length of the *line segment* joining the two points.

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| For horizontal and vertical *line segments*, the distance can be calculated by finding the difference between the *x- or y-coordinates*. | |
| Horizontal Line   * When a *line segment’s* ends, or two points, have the same *y-coordinate*, it is a horizontal line. * In this example, points *A* and *B* have the same *y-coordinate*. * The length of is the difference between the *x-coordinates*. |  |
| Vertical Line   * When a *line segment’s* ends, or two points, have the same x*-coordinate*, it is a vertical line. * In this example, points C and D have the same x-coordinates, so it is vertical. * The length of is the difference between the *y-coordinates*. |  |

* The *Pythagorean Theorem* can be used to find the distance between any two points in a coordinate plane when the *line segment* is neither horizontal nor vertical, representing a hypotenuse.



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| **Example**: Find the distance between the points and . | |
| **Step 1**: Graph the two points and draw a horizontal and vertical line to form a right triangle . |  |
| **Step 2**: Use what you know about horizontal and vertical line distance to calculate the length of *a* and *b*. | * and therefore * and therefore |
| **Step 3**: Substitute the values of a and b into *Pythagorean Theorem* to solve for the unknown length, *c*. |  |
| **Step 4**: State the answer. | The distance between *J* and *K* is 13. |

* There is a formula that can be used to determine the distance between any two points in the coordinate plane, called the *distance formula*, which is derived from *Pythagorean Theorem*.

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| The distance *d* between two points with the coordinates and is: |

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| **Example:** Use the distance formula to find the distance between and . | |
| **Step 1:** Identify the coordinate values. |  |
| **Step 2:** Substitute the values into the distance formula and solve for *d*. |  |
| **Step 3:** State the answer. | The distance between *L* and *M* is 15. |

**Objective 2:** In this section, you will use coordinates to compute the perimeter of polygons.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* If you know the *coordinates* of the *vertices* of a *polygon*, you can graph them on a coordinate plane, connecting them to form a *polygon*.
* You can determine the *perimeter* of a *polygon* by finding the sum of the lengths of the sides of the *polygon*.
* [Recall](#Bookmark1) how to find the distance, or lengths, of line segments using different methods, like the *distance formula* and *Pythagorean Theorem*.

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| **Example**: What is the perimeter of a pentagon *PQRST* with vertices , , , , and . | |
| **Step 1**: Plot the vertices on the coordinate plane and connect them to create a pentagon. |  |
| Use the distance formula to determine the lengths of each side of the pentagon *PQRST.* | |
| **Step 2**: Find the length of *PQ*. |  |
| **Step 3**: Find the length of *QR*. |  |
| **Step 4**: Find the length of *RS*. |  |
| **Step 5**: Find the length of *ST*. |  |
| **Step 6**: Find the length of *PT*. |  |
| **Step 7**: Add the lengths together to find the perimeter. |  |
| **Step 8**: State the answer. | The perimeter of pentagon *PQRST* is 19.7. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | *Use the image to answer the question.*  A straight, solid line connects plotted points upper A and upper B on a coordinate plane. The line slopes downward from left to right in quadrant 1. Point upper A is left parenthesis 1 comma 5 right parenthesis. Point upper B is left parenthesis 5 comma 2 right parenthesis.  What is the distance between the two points shown on the grid?  The distance between the two points is \_\_\_ units. | 5 |
| P 2 | Fatou marks the points (3, -5) and (-2, -7) on the coordinate plane. Then, she finds the distance between them. What is this distance rounded up to the nearest tenth of a unit?  The distance is about \_\_\_ units. | 5.4 |
| P 3 | *Use the image to answer the question.*  Triangle upper P upper Q upper R is graphed on a coordinate plane. The x-axis ranges from negative 8 to 4 in increments of 1. The y-axis ranges from negative 3 to 7 in increments of 1.  Use the given coordinates to compute the perimeter of the triangle. Round your answer to the nearest tenth. | 15.4 |
| P 4 | *Use the image to answer the question.*  Polygon upper A upper B upper C upper D upper E is graphed on a coordinate plane. The x-axis ranges from negative 3 to 3 in increments of 1. The y-axis ranges from negative 3 to 3 in increments of 1.  Using the coordinates, what is the perimeter of the polygon? Round each calculation to the nearest tenth. | 12 |
| P 5 | *Use the image to answer the question.*  Triangle upper A upper B upper C is graphed on a coordinate plane. The x-axis ranges from negative 3 to 3 in increments of 1. The y-axis ranges from negative 3 to 3 in increments of 1.  Using the given coordinates, what is the perimeter of the triangle? Round your answer to the nearest tenth.  The perimeter is approximately \_\_\_ units. | 11.1 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | *Use the image to answer the question.*  A straight, solid line connects plotted points E and F on a coordinate plane. The line slopes downward from left to right. Point E is left parenthesis negative 6 comma 1 right parenthesis. Point F is left parenthesis 2 comma negative 5 right parenthesis.  How would you set up the distance formula to find the distance between points *E* and *F.* |  |
| Q 2 | DaQuan marks two points on the coordinate plane. One point is *L* (4, 2) and the other point is *M* (7, 6). What is the correct way for DaQuan to set up the distance formula? |  |
| Q 3 | *Use the image to answer the question.*  Parallelogram upper A upper B upper C upper D is graphed on a coordinate plane. The x-axis ranges from negative 6 to 6 in increments of 1. The y-axis ranges from negative 6 to 6 in increments of 1.  Use the given coordinates to compute the perimeter of the parallelogram. | 26.6 units |
| Q 4 | *Use the image to answer the question.*  Trapezoid upper A upper B upper C upper D is graphed on a coordinate plane. The x-axis ranges from negative 1 to 6 in increments of 1.  The y-axis ranges from negative 1 to 4 in increments of 1.  Use the coordinates to compute the perimeter of the trapezoid. Round each calculation to the nearest tenth. | 14.4 units |
| Q 5 | *Use the image to answer the question.*  A triangle is graphed on a coordinate plane. The x-axis ranges from negative 2 to 6 in increments of 1. The y-axis ranges from negative 2 to 4 in increments of 1.  Use the coordinates to compute the perimeter of the triangle. | 11.2 units |

**Lesson 3 – Area in the Coordinate Plane**

**Key Words:**

* **altitude** – a line segment drawn from a vertex of a triangle perpendicular to the opposite side
* **area** – the amount of space taken up by a two-dimensional shape
* **circumscribe** – to draw a shape around a figure so that the shape intersects the vertices of the figure
* **composite figure** – a two-dimensional figure made up of multiple two-dimensional figures; a two-dimensional figure that can be divided into multiple two-dimensional figures
* **dimensions** – linear measures of distance for a shape (for example, length, width, height)
* **perpendicular** – intersecting at a 90-degree angle
* **rectangle** – a four-sided shape that has four right angles and two pairs of opposite sides that are parallel and congruent
* **trapezoid** – a four-sided polygon with exactly one pair of parallel sides

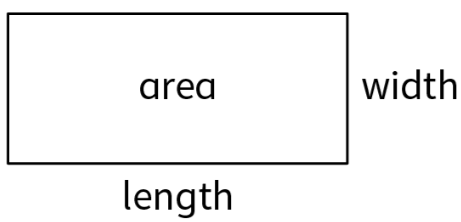
**Formulas:**

* Area of a Rectangle:
* Area of a Triangle:
* Area of a Trapezoid:
* Distance Formula:

**Objective 1:** In this section, you will use coordinates to compute areas of rectangles on a coordinate plane.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* [Recall](#Bookmark1) the distance formula, Pythagorean Theorem, and other methods that you can use to find the measures of the side lengths of polygons in a coordinate plane.
* Just as the side lengths can help you calculate the perimeter of a polygon; they can also help you calculate the *area* of a polygon.
* Recall that the area of a rectangle is the product of its length and width.
  + Area = length x width or
  + 

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| **Example**: What is the area of rectangle *WXYZ* with vertices , , , and ? | |
| **Step 1**: Start by graphing rectangle *WXYZ*. |  |
| **Step 2**: Use the distance formula to calculate the width and the length of the rectangle. Remember, you only need the two sides of a rectangle (a length and a width) to calculate the area. | For rectangle *WXYZ*, and . You only need to calculate the distance of and . |
| **Step 3**: Use the dimensions to calculate the area using the formula for a rectangle area. |  |
| **Step 4:** State the answer. | The area of rectangle *WXYZ* is 11.8 square units. |

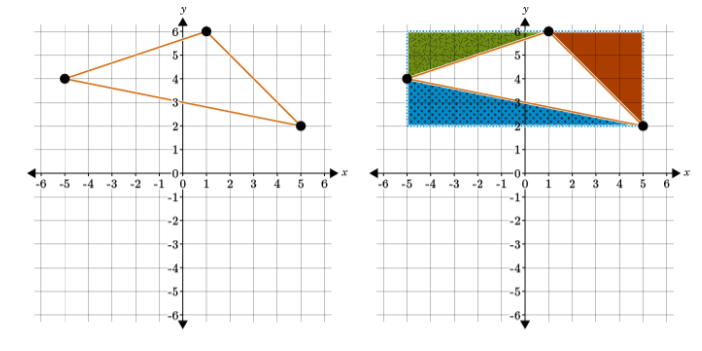
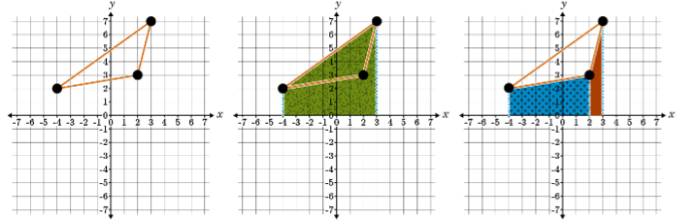
**Objective 2:** In this section, you will use coordinates to compute the areas of triangles in a coordinate plane.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* [Recall](#Bookmark1) the distance formula, Pythagorean Theorem, and other methods that you can use to find the measures of the side lengths of polygons in a coordinate plane.
* Just as the side lengths can help you calculate the perimeter of a polygon; they can also help you calculate the *area* of a polygon.
* Recall that the area of a *triangle* is half the product of its base and its height.
  + or
* Sometimes, the height is not obvious, and you must determine it by finding the length of the *altitude*.
  + The *altitude* is the line segment drawn from the vertex opposite the base and *perpendicular* to the base.

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| **Example**: What is the area of with vertices , , and ? | |
| **Step 1**: Graph the vertices on the coordinate plane and connect them to create a triangle. |  |
| **Step 2**: Find the length of the base of the triangle. | Segment *KL* is a vertical line segment, so you can consider that the base. |
| **Step 3**: Find the height, or altitude, of the triangle. | Draw the altitude from the vertex that is opposite and perpendicular to the base and calculate the distance. |
| **Step 4**: Calculate the area using the formula for a triangle's area. |  |
| **Step 5:** State the answer. | The area of is 12 square units. |

* There are other methods for finding the area of triangles using composite figures.
  + For acute triangles, when no sides are horizontal or vertical line segments, you can use *rectangles*.
  + The *area* of the acute triangle is the area of the *rectangle* minus the sum of the areas of the right triangles.
  + 
* You can also use *trapezoids* to create a composite figure that includes the triangle.
  + The *area* of an obtuse triangle is the largest trapezoid minus the sum of the two smaller trapezoids.
  + 

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | *Use the image to answer the question.*  Rectangle upper A upper B upper C upper D is graphed on a coordinate plane. The x-axis ranges from negative 5 to 5 in increments of 1 The y-axis ranges from negative 5 to 5 in increments of 1.  Determine the coordinates of rectangle *ABCD*. Use the coordinates to compute the area of the rectangle.  \_\_\_ units2 | 21 |
| P 2 | *Use the image to answer the question.*  Rectangle upper A upper B upper C upper D is graphed on a coordinate plane. The x-axis ranges from negative 2 to 18 in increments of 1. The y-axis ranges from negative 2 to 18 in increments of 1.  Use the coordinates to compute the approximate area of the rectangle. Round your answer to the nearest whole number.  \_\_\_ units2 | 79 or 80 |
| P 3 | *Use the image to answer the question.*  A triangle is graphed by joining three plotted points on the first quadrant of a coordinate plane. The x and y axes range from 0 to 10 in increments of 1.  Fiona draws a triangle on a coordinate grid with vertices at (6, 2), (4, 6), and (1, 2). What is the area in square units of Fiona’s triangle?  The area of the triangle is \_\_\_\_ square units. | 10 |
| P 4 | *Use the image to answer the question.*  A triangle is graphed by joining three plotted points on the first quadrant of a coordinate plane. The x and y axes range from 0 to 10 in increments of 2.  Luella drew the triangle shown on a coordinate grid. To find the triangle’s area, she multiplies ½ by 2 and then multiplies the result by another number. What is the number Luella can use to complete the calculation?  Luella can multiply ½ by 2 by \_\_\_\_\_ to find the triangle’s area. | 8 |
| P 5 | *Use the image to answer the question.*  A triangle is graphed by joining three plotted points on a coordinate plane. The x and y axes range from negative 10 to 10 in increments of 2.  Leonardo draws the obtuse triangle pictured here and calculates the area in square units. What is the area of Leonardo’s triangle?  Leonardo’s triangle has an area of \_\_\_\_ square units. | 20 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | *Use the image to answer the question.*  Rectangle upper A upper B upper C upper D is graphed on a coordinate plane. The x-axis ranges from negative 7 to 2 in increments of 1. The y-axis ranges from negative 1 to 8 in increments of 1.  Find the coordinates of the rectangle and use them to compute the area. | 15 units2 |
| Q 2 | *Use the image to answer the question.*  Rectangle upper A upper B upper C upper D is graphed on a coordinate plane. The x-axis ranges from negative 2 to 8 in increments of 1. The y-axis ranges from negative 5 to 4 in increments of 1.  Use the coordinates to compute the exact area of the rectangle. | 24 units2 |
| Q 3 | *Use the image to answer the question.*  Rectangle upper A upper B upper C upper D is graphed on a coordinate plane. The x-axis ranges from negative 5 to 5 in increments of 1. The y-axis ranges from negative 4 to 6 in increments of 1.  Determine the coordinates of the rectangle, then compute the area. | 26 units2 |
| Q 4 | *Use the image to answer the question.*  A triangle is graphed by joining three plotted points on the first quadrant of a coordinate plane. The x and y axes range from 0 to 10 in increments of 1.  Compute the area of the triangle using coordinates. What is the area of the triangle in square units? | 7 |
| Q 5 | *Use the image to answer the question.*  A triangle is graphed by joining three plotted points on the first quadrant of a coordinate plane. The x and y axes range from 0 to 10 in increments of 2.  Quincy draws this image of a triangle on the coordinate grid. Which of the following triangles has the same area as Quincy’s triangle? | A triangle is graphed by joining three plotted points on the first quadrant of a coordinate plane. The x and y axes range from 0 to 10 in increments of 1. |

**Lesson 4 – The Midpoint Formula**

**Key Words:**

* **average** – the total of all the values divided by the number of values
* **congruent** – of the same shape and size; in geometry, congruent parts overlap perfectly when placed on top of one another
* **coordinate** – a value that describes the exact position of a point on a number line
* **midpoint** – a point that is the exact middle of a line segment
* **x-coordinate** – the first coordinate in an ordered pair that tells the distance to travel left or right from the origin
* **y-coordinate** – the second coordinate in an ordered pair that tells the distance to travel up or down from the origin

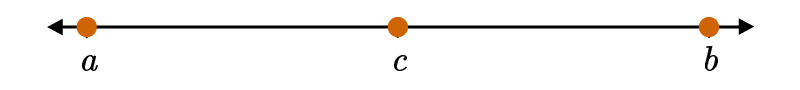
**Formulas:**

* Midpoint Formula:

**Objective 1:** In this section, you will derive the formula for finding the midpoint of a line segment in a coordinate plane.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas**:

* The *midpoint* of a line segment is halfway between the two endpoints and divides the segment into two *congruent* segments.
  + For example, if *c* is the *midpoint* and *a* and *b* are the end points, then you can find the *midpoint*, *c*, by averaging the endpoints:
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| **Example:** A line segment has endpoints and . What is the midpoint of segment ? |
| Find the average of the endpoints by using .  The midpoint of segment is –4.05. |

**Objective 2:** In this section, you will calculate the midpoint of a line segment in the coordinate plane.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

* A line segment on the coordinate plane has endpoints whose coordinates are an ordered pair *(x, y)*.
* The coordinate of the *midpoint* is the *average* of the coordinates of the endpoints.
  + The *x-coordinate* of the *midpoint* of a line segment is the average of the *x-coordinates* of its endpoints.
  + The *y-coordinate* of the *midpoint* of a line segment is the average of the *y-coordinates* of its endpoints.

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| **Midpoint Formula**  The midpoint M of a line segment with endpoints and is: |
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| **Example:** Segment has endpoint and endpoint . Find the coordinates of the midpoint, *M*, of *ST*. | |
| **Step 1:** Recall that you need two coordinate points to represent the end points. |  |
| **Step 2:** Calculate the average of the *x-*coordinates. | The *x-*coordinate of midpoint *M* is –1. |
| **Step 3:** Calculate the average of the *y-*coordinates. | The *y-*coordinate of midpoint *M* is 5. |
| **Step 4:** State the answer. | The coordinates of midpoint *M* are **.** |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | *Use the image to answer the question.*  A number line ranging from negative 1 to 15 shows a closed circle at 5 labeled upper M and a closed circle on 9 labeled upper B.  Given that *A* is the midpoint of line segment , find the value of *A*.  Point *A* lies at \_\_\_\_\_ on the number line. | 7 |
| P 2 | *Use the image to answer the question.*  Line segment upper A upper B is plotted on a coordinate plane with x-axis ranging from negative 5 to 4 and y-axis ranging from negative 3 to 5 both in increments of 1.  Find the midpoint *M* of the line segment with endpoints *A* (-3.2, 3.5) and *B* (2.1, -2.3). Enter your response as decimals. If the solution includes a decimal, do not round.  The midpoint of segment is (\_\_\_\_\_, \_\_\_\_\_). | -0.55; 0.6 |
| P 3 | *Use the image to answer the question.*   A line segment on a coordinate plane. Both the x-and y-axes range from negative 3 to 4 in unit increments. The line segment descends from upper A on the top left to upper B on the bottom right.  Find midpoint *M* of the line segment with endpoints *A* (-1.4, 2.5) and *B* (2.6, -1.5). Enter both responses as a decimal value to the tenths place.  The midpoint of segment is (\_\_\_\_\_, \_\_\_\_\_). | 0.6; 0.5 |
| P 4 | *Use the image to answer the question.*    A line segment on a coordinate plane. The x-axis ranges from negative 3 to 4 in unit increments. The y-axis ranges from negative 3 to 3 in unit increments. The line extends upward from point upper R on the bottom left, to point upper Q on the top right.  Yolanda calculated the midpoint of line segment to be (-2, 2). What mistake did she make?  Statement #1: Yolanda averaged the values from point *R* and then averaged the values from point *Q*.  Statement #2: Yolanda forgot to divide the sums by 2.  Statement #3: Yolanda subtracted when she should have added.  Statement # explains Yolanda’s mistake. | 1 |
| P 5 | *Use the image to answer the question.*   A line segment on a coordinate plane. The x-axis ranges from negative 1 to 6 in unit increments. The y-axis ranges from negative 3 to 4 in unit increments. The line extends downward from point upper J on the top left, to point upper K on the bottom right.  Find midpoint *M* of the line segment with endpoints *J* (1.2, 3.4) and *K* (4.8, -2.3). If the solution includes a decimal, do not round.  The midpoint of segment is at (\_\_\_, \_\_\_) | 3; 0.55 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | *Use the image to answer the question.*  Line segment upper A upper B is plotted on a coordinate plane. The x-axis ranges from 0 to 7 and the y-axis ranges from negative 3 to 7 both in one unit increments.  Using this graph, how would adding 6+2 then dividing by 2 help you find the midpoint of line ? | It finds the average distance between the two *x*-values of points *A* and *B*. |
| Q 2 | *Use the image to answer the question.*  Line segment upper J upper K slants down left to right on a coordinate plane. The x-axis ranges from negative 7 to 5 and y-axis ranges from 0 to 8 both in 1 unit increments.  Does adding 7+2 and then dividing by 2 help you find the midpoint of line segment ? | Yes because calculates the average, or midpoint, of the *y*-coordinates *J* and *K*. |
| Q 3 | *Use the image to answer the question.*  Line segment upper A upper B slants up left to right on a coordinate plane. The x-axis ranges from negative 5 to 8 and the y-axis ranges from negative 5 to 4 both in 1 unit increments.  To find the midpoint of line segment , Fernando calculated 6.4 + (-2.5) = 3.9. What should his next step be? | Fernando should divide 3.9 by 2, then repeat both steps using the *y*-values. |
| Q 4 | *Use the image to answer the question.*  A line segment on a coordinate plane. The x-axis ranges from negative 2 to 5 in unit increments. The y-axis ranges from negative 3 to 3 in unit increments. The line extends downward from point upper A on the top left, to point upper B on the bottom right.  Calculate midpoint *M* of segment . | *M* is located at (1.5, 0). |
| Q 5 | *Use the image to answer the question.*  A line segment on a coordinate plane. The x-axis ranges from negative 4 to 0 in unit increments. The y-axis ranges from negative 3 to 5 in unit increments. The line extends straight down from point upper A to point upper M.  If *R* is the midpoint of segment , find midpoint *R*. | *R* is located at (-2, 2.5). |

**Lesson 5 – Coordinate Proofs**

**Key Words:**

* **congruent** – of the same shape and size; in geometry, congruent parts overlap perfectly when placed on top of one another
* **coordinates** – numbers that describe the position of points along certain dimensions
* **equilateral** – having all sides equal
* **isosceles** – having two congruent sides
* **parallel lines** – a pair of lines that never intersect and have the same slope
* **perpendicular lines** – two lines that have opposite reciprocal slopes
* **radius** – line from the center to the circumference of a circle
* **right angle** – an angle whose measure is 90 degrees
* **scalene** – having no sides equal in length
* **slope** – the measure of the steepness of a line
* **slope-intercept form of a line** – the equation y=mx+b, where m is the slope of the line and b is the y-intercept
* **y-intercept** – a point where the graph of a line crosses the y-axis

**Formulas:**

* Distance Formula:
* Slope Formula:

**Objective 1:** In this section, you will use coordinates in geometry proofs.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* You can use the [distance formula](#Bookmark2) in geometry proofs involving triangles, squares, and circles.
* Recall the following about the different geometric shapes:
  + A *scalene* triangle has no sides that are equal in length.
    - Calculate the length of each side of the triangle. If they are all different lengths, it is a scalene triangle.
  + An isosceles triangle has two legs of the same length and one different leg.
    - Calculate the length of each side length. If two of the distances are the same, then the triangle is an *isosceles* triangle.
  + All four sides of a square are equal in length.
    - Calculate the distance of each segment that makes up a square. If all four distances are equal, it is a square.
  + Every point on a circle is the same distance from its center.
    - Calculate the distance of a given point from the center of a circle. If the distance is the same as the *radius*, then the point is on the circle.

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| **Example**: Prove that the points , , and form an isosceles triangle (). | |
| **Step 1**: Use the distance formula to calculate the distance of line segment . |  |
| **Step 2**: Use the distance formula to calculate the distance of line segment . |  |
| **Step 3**: Use the distance formula to calculate the distance of line segment . |  |
| **Step 4**: State the answer. | has two congruent sides since . Thus, is isosceles. |

**Objective 2:** In this section, you will solve geometric problems by calculating the slope of parallel and perpendicular lines.

*Mathematical Practice Standard: You will look for and make use of structure.*

**Big Ideas:**

* Recall the properties of *parallel* and *perpendicular lines*.

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| Parallel Lines   * *Parallel lines* never cross because they have the same *slope*, meaning they have the same steepness on a graph. * *Parallel lines* always have a different value for the *y-intercept*; otherwise, they would be the same line. | For example: and |
| Perpendicular Lines   * Recall that *perpendicular lines* have *slopes* which are negative reciprocals so that they intersect at *right angles*. | For example: and  \*and any other line with a slope of |

* Recall how to calculate the *slope* of a line.
  + Use the coordinates of any two points on a line, respectively, and .
  + Slope formula:
* *Parallel and Perpendicular lines* help prove the classification of quadrilaterals. Recall the properties of different quadrilaterals:

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| **Properties of Trapezoids** | * one pair of parallel sides |
| **Properties of Kites** | * no parallel sides * diagonals are perpendicular * one diagonal bisects the opposite angle |
| **Properties of Parallelograms** | * opposite sides are parallel * opposite sides are congruent * opposite angles are congruent * consecutive angles are supplementary |
| **Properties of a Rectangle** | * opposite sides are congruent * opposite sides are parallel * all four angles are equal * opposite angles are congruent * all angles are right angles * diagonals are congruent |
| **Properties of a Rhombus** | * all four sides are congruent * opposite sides are parallel * opposite angles are congruent * diagonals are perpendicular |
| **Properties of a Square** | * all four sides are congruent * opposite sides are congruent * opposite sides are parallel * opposite angles are congruent * all angles are right angles |

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| **Example:** Quadrilateral has vertices at , , , and . Prove that is a parallelogram. | |
| **Step 1:** Recall the properties of a parallelogram. | Parallelograms have two pairs of parallel lines.  and must have the same slope to be parallel.  and must also have the same slope to be parallel. |
| **Step 2:** Use the slope formula to calculate the slope of . |  |
| **Step 3:** Use the slope formula to calculate the slope of . |  |
| **Step 4:** Determine if and are parallel. | and are parallel because they have the same slope. |
| **Step 5:** Use the slope formula to calculate the slope of . |  |
| **Step 6:** Use the slope formula to calculate the slope of . |  |
| **Step 7:** Determine if and are parallel. | and are parallel because they have the same slope. |
| **Step 8:** State the answer. | Since has two sets of parallel lines, we can state that is a parallelogram. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | The center of circle C is at (6, 2), with a radius of 5. In order to prove that point D (6, -3) lies on the circle, what does the length of segment need to be?  Segment must have a length of \_\_\_\_\_. | 5 |
| P 2 | Amy is trying to prove that a triangle with the vertices X (-6, 2), Y (-4, 6), and Z (-2, 2) is an isosceles triangle. She found that side has a length of approximately 4.5 and that side has a length of 4. What is the length of side ? Round your answer to one decimal place.  Side has a length of \_\_\_\_. | 4.5 |
| P 3 | *Use the image to answer the question.*  Two points are plotted on a coordinate plane, and a line is graphed passing through them. The x-axis ranges from negative 7 to 7 in increments of 1. The y-axis ranges from negative 7 to 7 in increments of 1.  Calculate the slope of a line that is perpendicular to the one shown in the graph.  *m* = \_\_\_\_\_ |  |
| P 4 | Line segment on parallelogram ABCD has a slope of -3. What is the slope of line segment ?    *m* = \_\_\_\_\_ | -3 |
| P 5 | Ruben is designing an image using computer software. A grid that looks like a coordinate plane is his drawing board. He needs to construct a line parallel to an existing line . What is the slope of the parallel line?  *m* = \_\_\_\_\_ | -5 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | *Use the image to answer the question.*  A coordinate plane's x-axis ranges from negative 6 to 1 and its y-axis ranges from negative 2 to 6, both by 1-unit increments. A triangle made up of vertices upper A, upper B, and upper C is plotted.  Use a coordinate proof to show that triangle *ABC* is scalene. What are the lengths of sides *AB*, *BC*, and *AC*? | ; therefore, *ABC* is a scalene triangle. |
| Q 2 | *Use the image to answer the question.*  A coordinate plane's axes range from 0 to 8, both by 1-unit increments. A point labeled upper A is plotted at left parenthesis 4 comma 4 right parenthesis. A circle is centered around this point.  The center of circle *A* is at (4, 4), with a radius of 3. What should Negin’s first step be to prove that the point *B* (7, 4) lies on the circle? | Negin should use the distance formula to calculate the length of segment *AB*. |
| Q 3 | *Use the image to answer the question.*  Two points are plotted on a coordinate plane, and a line is graphed passing through them. The x-axis ranges from negative 10 to 5 in increments of 1. The y-axis ranges from negative 4 to 8 in increments of 1.  Calculate the equation of a line that is perpendicular to the line shown on the graph and that also shares the same *y*-intercept. |  |
| Q 4 | Mallory is designing a background print for a logo. She starts the design on a coordinate plane and creates parallel lines diagonally. The first line that she makes has a slope of and crosses through the origin. If she wants the next line to cross the *y*-axis at 2, what would be the equation of the next parallel line? |  |
| Q 5 | Which equation fits a line that is parallel to the line ? |  |