Algebra 1

**Modeling with Algebra**

**Unit Summary:** In this unit, you will make use of linear, exponential, and quadratic functions to model the relationship between two variables.

**Lesson 2 – Creating a Model from a Graph**

**Key Words:**

* **function** – an expression, rule, or law that defines a relationship between the independent variable and the dependent variable
* **graph** – a visual diagram that shows the relationship between two quantities
* **linear function** – an equation in the form , where *m* is the slope and *b* is the *y*-intercept

**Formulas:**

* Linear Function:
* Slope Formula:

**Objective 1:** In this section, you willinterpret key features of contextual function graphs to identify their types.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* Recall that a *function* is an expression of the relationship between two quantities.
* A *function* can also be expressed as a graph to show the functions behavior.
* You can use the *graph* of a *function* to identify key features and other information about the *function*.

|  |  |
| --- | --- |
| **Example:** When an object is thrown upward from the ground with an initial velocity of 24 feet per second its height is modeled by the function . The graph of the function is given here. Use the graph to answer questions about the key features of the function. | |
| **What is the maximum height of the object?** | The graph represents the height of the object over time, so the maximum height of the object corresponds to the maximum of the graph, which is 4.5 feet. |
| **During which periods of time is the object going up and falling down?** | The time period during which the object is going up corresponds to the increasing interval of the graph, which is 0 to 0.375.    The time period during which the object is falling corresponds to the decreasing interval of the graph, which is 0.375 to 0.75. |
| **After how many seconds does the object reach the ground?** | When the object reaches the ground, its height is 0. This corresponds to the x-intercepts of the graph.  There are two x-intercepts, one at when the object is initially thrown. The other is at when the ball has landed.  Therefore, the object reaches the ground at 0.75 seconds. |

**Objective 2:** In this section, you will create a table and equation to use as a model when given a contextual graph.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* *Functions* can be represented as equations, tables, or graphs.
* The three representations of one *function* are equivalent and when given one form of the *function*, you can figure out the remaining forms.

|  |
| --- |
| **Linear Function** |
| *m* is the slope  *b* is the *y*-intercept |
| The graph of a linear function is a straight line. |
| The slope of the function, *m*, can be found using the formula:  where and are two given points of the function |

|  |  |
| --- | --- |
| **Example:** The following graph represents the cost of garlic in pounds.   1. Create a table with at least three function values. 2. Create an equation to model the graph. | |
| **Create a table with at least three function values.** | * Investigate the graph and select three points that are located precisely on the grid points. * Three points: (0, 0), (2, 9), (4, 18) * Write the points in a table:  |  |  | | --- | --- | | ***x*** | ***f(x)*** | | 0 | 0 | | 2 | 9 | | 4 | 18 | |
| **Create an equation to model the graph.** | * The given graph is a straight line; therefore, the model will be a linear function . * Use the given graph to identify the *y-*intercept, *b*.   + The *y*-intercept is at (0,0), therefore, * Use the two points from the table and the slope formula to find the slope, *m*. * Use the values of m and b to create the linear function. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Use the image to answer the question.  A curve is plotted on a coordinate plane, representing distance and time.  Mike is driving home from school for Thanksgiving. Shown here is the graph of the distance from Mike’s home to his school as a function of time. How many miles from home is Mike’s school?  The distance from Mike’s home to his school is \_\_\_ miles. | 100 |
| P 2 | Use the image to answer the question.  A coordinate plane's x-axis ranges from 0 to 80 by 10-unit increments and the y-axis ranges from negative 300 to 1,000 by 100-unit increments. The x-axis is labeled 'Number of Heaters Sold' while the y-axis is labeled 'Profit' with dollar units.  A local store is selling heaters. The graph represents the profit made as a function of the number of heaters sold. How many heaters does the store need to sell to make a profit of $700?  The store needs to sell \_\_\_ heaters to make a profit of $700. | 40 |
| P 3 | Use the image to answer the question.  A ray with an arrow on the top end extends from the origin on the first quadrant of a coordinate plane. The x-axis for Time in hours ranges from 0 to 5 in increments of 1 and the y-axis for Distance in miles ranges from 0 to 240 in increments of 20.  The graph shows the total distance traveled by a car driving at a constant speed of 60 miles per hour. Based on the information from the graph, complete the table as shown here. | 1. 60 2. 3 3. 240 |
| P 4 | Use the image to answer the question.  A ray with an arrow on the top end extends upward on the first quadrant of a coordinate plane. The x-axis for Distance Traveled in miles ranges from 0 to 25 in increments of 5 and the y-axis for Rental Cost in dollars ranges from 0 to 30 in increments of 5.  The graph represents the cost of renting a car based on the distance traveled. Find the equation of the linear function that models this graph. | 1. 0.25 2. 20 |
| P 5 | Use the image to answer the question.  A downward-opening parabola is graphed on a coordinate plane. The x-axis for Width ranges from negative 2 to 10 in increments of 2 and the y-axis for Area ranges from negative 2 to 18 in increments of 2.  Given is the graph representing the area of a rectangle whose perimeter is 16. Using the graph, complete the table as shown here. | 1. 0 2. 4 3. 7 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Use the image to answer the question.  A coordinate plane's axes range from 0 to 60 by 10-unit increments. The x-axis is labeled 'Time' with minute units and the y-axis is labeled 'Distance' with mile units. A curve is plotted.  Rania is heading home from work. Along the way, she stops at the grocery store to buy some fresh vegetables. Shown here is the graph of the distance from Rania’s home to her workplace. Interpret the graph. How long does it take Rania to get home? | 55 minutes |
| Q 2 | Use the image to answer the question.  A coordinate plane's x-axis ranges from 0 to 300 by 50-unit increments and the y-axis ranges from 0 to 12,000 by 2,000-unit increments. The x-axis is labeled 'Number of Laptops Sold' and the y-axis is labeled 'Revenue' with dollar units.  SmartComputer makes and sells laptops. The graph here shows the revenue of SmartComputer as a function of the number of laptops sold. Interpret the graph. How much revenue is earned if 75 laptops are sold? | $6,000 |
| Q 3 | Use the image to answer the question.  A line graphed in the first quadrant of a coordinate plane shows number of campers versus total cost in dollars.  Quinn’s family is going camping. To get into a camping site, his family must pay a one-time parking fee and purchase a ticket for each family member. The graph shows the total cost Quinn’s family must pay to enter. Which of the following points can be used to find the linear equation that models this graph? | (0, 25) and (5, 60) |
| Q 4 | Use the image to answer the question.  A curve is plotted in the first quadrant of a coordinate plane. The x-axis for Time in seconds ranges from 0 to 1.25 in increments of 0.25 and the y-axis for Height in feet ranges from 0 to 18 in increments of 2.  The graph here represents the height of a ball when being dropped from a height of 16 feet. Which of the following points can be used to find the quadratic equation that models the graph? | (0, 16), (0.5, 12), and (1, 0) |
| Q 5 | Use the image to answer the question.  A downward-opening parabola is graphed on a coordinate plane. The x-axis for Width ranges from negative 2 to 10 in increments of 2 and the y-axis for Area ranges from negative 2 to 18 in increments of 2.  The graph represents the area of a rectangle whose perimeter is 16. The graph is modeled by a quadratic equation of the form . Given that c=0, using the points (3,15) and (6,12), which system of linear equations can be used to identify the coefficients a and b? | and |

**Lesson 3 – Using a Model from a Graph**

**Key Words:**

* **function** – a relation where each input is assigned to exactly one output
* **graph** – a visual diagram that shows the relationship between two quantities

**Formulas:**

* Linear Function:

**Objective 1:** In this section, you will interpret a function graph and model in terms of the context.

*Mathematical Practice Standard: Use appropriate tools strategically.*

**Big Ideas**:

* A *function* can be used to model a real-life situation.
* When an equation is given, you can use the *function* equation to solve for a desired quantity.

|  |  |
| --- | --- |
| **Example:** The function represents the monthly cost in dollars of a phone plan where *x* is the number of minutes used that month. | |
| **What do the numbers 0.03 and 12 represent in the function?** | * 0.03 is the rate in dollars per minute.   + Customers are charged $0.03 per minute. * 12 represents a flat monthly fee.   + If a customer uses 0 minutes, they are still charged $12. |
| **What is and what does it represent?** | * When a customer uses 235 minutes in a given month, they are charged $19.05. |
| **If , what is x and what does it represent?** | * Substitute into the function . * Solve for x.   + Subtract 12 from both sides:   + Divide both sides by 0.03: * 423 minutes were used in a month where the bill was $24.69. |

* When a *graph* is given, you can use the function *graph* to identify desired quantities.
  + Pay attention to the labeling of the *x-*axis and the *y-*axis and identify what each axis stands for in the context of the problem.

|  |  |
| --- | --- |
| **Example:** In 2007, Best Cookies Bakery introduced a new cookie flavor. Given is the graph of the function representing the price for each bag of this new cookie flavor over time since 2007.    In what years was the price of a bag of cookies $6? | |
| **Step 1:** Identify the variables using the graphs axis. | *x* = time (years)  *y* = price per bag of cookies (dollars) |
| **Step 2:** Locate points on the graph with a *y*-coordinate of 6, to represent $6 dollars. | The points (2,6), (4,6), and (6,6) all have a *y-*coordinate of 6. |
| **Step 3:** Identify which years correspond to the *x-*values. | *x* represents the number of years after 2007:   * x = 2: 2009 * x = 4: 2011 * x = 6: 2013 |
| **Step 4:** State the answer. | The price of a bag of cookies was $6 in the years 2009, 2011, and 2013. |

**Objective 2:** In this section, you will report graphical analysis results with an appropriate level of precision.

*Mathematical Practice Standard: Attend to precision.*

**Big Ideas:**

* When analyzing the *graph* of a *function*, it is sometimes difficult to determine an exact value of a quantity, or a realistic value of a real-life situation.

|  |  |
| --- | --- |
| **Example:** Given is the graph representing the total distance traveled by a car over time at a constant speed. Based on the graph, which of the following options is a good estimate Graphing Calculator for the total distance traveled in 1.5 hours with appropriate level of precision?   1. 60 miles 2. 89 miles 3. 92.53 miles | |
| **Step 1:** Understand the axes. | * The *x*-axis shows time (in hours). * The *y*-axis shows distance (in miles). |
| **Step 2:** Locate the *x-*value on the graph and its corresponding *y*-value. | The y-value tells you the total distance traveled at 1.5 hours. It is between 80 and 100 miles. |
| **Step 3:** Eliminate unrealistic answer choices to identity the answer. | Recall the answer choices:   1. 60 miles 2. 89 miles 3. 92.53 miles  * If an answer is below 80 (like 60), it’s **too low**. * If an answer is too exact (like 92.53), it’s **too precise** for this estimate. * A reasonable and properly rounded estimate is **89 miles**. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Consider the linear function  for the following context. Sandra starts off with $150 in her bank account. If she saves $25 every week (where x represents the number of weeks that she has been saving money), how much would Sandra have in her account after 14 weeks? | 500 |
| P 2 | Use the image to answer the question.  A curve showing the price of a bag of cookies over time is plotted on a coordinate plane. The y-axis is labeled price per bag of cookies left parenthesis dollars right parenthesis. The x-axis is labeled time left parenthesis years right parenthesis.  In 2007, Best Cookies Bakery introduced a new cookie flavor. The following is the graph of the function representing the price for each bag of this new cookie flavor since 2007. Interpret the following function graph to determine the year that the price per bag of cookies was $4. | 2008 |
| P 3 | Apply the linear function  to the following context. A large cheese pizza costs $9, and each additional topping costs $0.75. In this function,*x* represents the number of toppings that you choose to put on your pizza. Determine the number of toppings you chose to put on your pizza if the total cost was $13.50. | 6 |
| P 4 | Use the image to answer the question.  Four plotted points are joined by lines on the first quadrant of a coordinate axis. The x-axis is labeled as season with values as spring, summer, fall, and winter. The y-axis is labeled as price in dollars from 0 to 13 in unit increments.  The graph shows the average seasonal price of a four-count bag of lemons. Use the graph to report the average price of one lemon in the summer. | 1.25 |
| P 5 | Safina measured the perimeter of a shoebox to be 24 inches. Report the minimum and maximum possible perimeter based on the appropriate level of precision. Enter your responses in decimal form. | 1. 23.5 2. 24.5 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Use the image to answer the question.  An open, downward facing parabola is graphed on a coordinate plane. The x-axis ranges from negative 2 to 5 in increments of 1. The y-axis ranges from negative 1 to 16 in increments of 2.  Interpret the following quadratic function model and graph given the following context. A volleyball is served into the air at 26 ft./sec from a height of 4.5 ft. The quadratic equation represents the height of the ball over time and in seconds. The graph illustrates this path where x represents the time in seconds and f(x) represents the height in feet. Approximately how long does it take for the volleyball to reach maximum height? | approximately 0.85 seconds |
| Q 2 | Use the image to answer the question.  A curve showing an increase of weight over time is plotted on a coordinate plane. The y-axis is labeled weight left parenthesis pounds right parenthesis. The x-axis is labeled time left parenthesis months right parenthesis.  Interpret the following exponential function graph to determine the approximate weight of the child when they are 6 months old. | 17 pounds |
| Q 3 | Interpret the following linear function model with the given context. Leslie wants to throw a birthday party at their local bowling alley. The space is $100 to rent with an additional charge of $15 per person attending. This scenario of total cost is modeled by the linear function . Leslie gets a bill of $550 at the end of the evening. In addition to Leslie and Steve, how many other people attended this birthday party? | 28 |
| Q 4 | Report the longest possible length with the appropriate level of precision for a measurement of 34.2 meters. | 34.25 |
| Q 5 | Use the image to answer the question.  A rectangle has a length labeled 5.2 inches and width labeled 3.8 inches.  Which response represents the longest possible length and width of the rectangle with the appropriate level of precision? | 3.85 and 5.25 inches |

**Lesson 4 – Creating a Model from a Sequence**

**Key Words:**

* **arithmetic sequence** – a sequence (such as 1, 3, 5) in which the difference between a term and its predecessor is always the same
* **common difference** – the difference between two consecutive terms of an arithmetic progression
* **common ratio** – the ratio of each term of a geometric progression to its preceding term
* **explicit formula** – a sequence in which the terms are defined by using the position of the term
* **function** – an expression, rule, or law that defines a relationship between the independent variable and the dependent variable
* **geometric sequence** – a sequence (such as 1, 12, 14) in which the ratio of a term to its predecessor is always the same
* **recursive formula** – a formula expressing any term of a sequence as a function of one or more preceding terms
* **sequence** – a set of numbers that follow a specific pattern or formula
* **term** – is either a single number or variable, or numbers and variables multiplied together

**Formulas:**

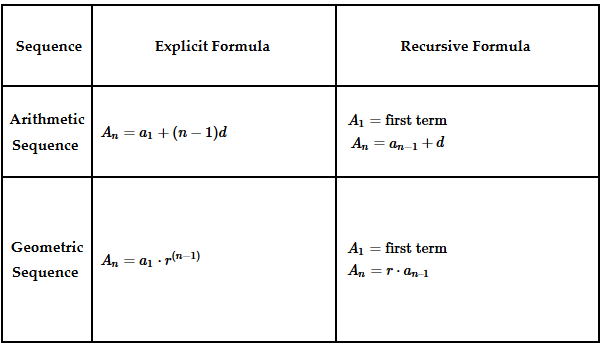
* Arithmetic Sequence:
  + Explicit Formula:
    - As a function:
  + Recursive Formula: ,
* Geometric Sequence:
  + Explicit Formula:
    - As a function:
  + Recursive Formula: ,

**Objective 1:** In this section, you will identify a given sequence in context as arithmetic, geometric, or another type.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas**:

* A *sequence* is a set of numbers that follow a specific pattern or formula.
* *Arithmetic sequences* have a *common difference* between terms.
  + This difference is called the *common difference* (*d*).
  + Example: The sequence 15, 17, 19, 23, ... has a *common difference* of .
  + How to check: check if each term increases or decreases by the **same amount**.
* *Geometric sequences* have a constant *ratio* between terms.
  + This ratio is called the *common ratio* (*r*).
  + Example: The sequence 100, 50, 25, 12.5, ... has a *common ratio* of .
  + How to check: check if each term is multiplied or divided by the **same number**.
* *Sequences* can be written using an *explicit* or *recursive formula.* 
  + These formulas are helpful when trying to find the values of specific terms of a sequence.
  + The terms of a sequence are represented by the variable *n*.
  + For example: In the sequence 100, 50, 25, 12.5, ..., the number 50 is the second term of the sequence, or , because it has the second position in the sequence.



|  |  |
| --- | --- |
| **Example:** Kurt is trying to reduce his daily screen time. Currently, he spends an average of 4 hours each day in front of a TV, laptop, or phone screen. His plan is to reduce his screen time by 15 minutes each week.   1. Identify the type of sequence described. 2. Find the explicit expression for the sequence. 3. If his plan goes accordingly, how much time will Kurt spend in front of a screen in the fifth week? | |
| **Step 1:** Write the sequence being described. | Currently, Kurt spends 4 hours, or 240 minutes of screen time each day.   * The first term of the sequences is   After one week, he will reduce his screen time by 15 minutes.   * The second term of the sequence is   After two weeks, he will reduce his screen time by another 15 minutes.   * The third term of the sequence is   The pattern continues, and the sequence of Kurt’s screen time is 240, 225, 210, ... |
| **Step 2:** Identify the type of sequence. | * Notice that the difference between any two consecutive terms is –15. * This is an arithmetic sequence with a common difference of . |
| **Step 3:** Write the explicit formula of the sequence. | * Explicit formula: * Identify the variables: , * Write explicit expression: or |
| **Step 4:** Use the explicit formula to calculate how much Kurt’s screen time will be on the fifth week. | * Explicit formulas: * Find the value in the fifth week, , by substituting 5 for *n a*nd solving. * Kurt will spend 180 minutes in front of a screen in the fifth week. |

**Objective 2:** In this section, you will create a function to use as a model of a given sequence in context.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* *Sequences* can be represented by a *function* that models the *sequence*.
* The function modeling an arithmetic and geometric sequence **shares the same expression with its explicit formula.**
* [Recall](#Bookmark1) the *explicit* *formulas* for *arithmetic* and *geometric* sequences. Notice how they are written when modeling a function.
  + Arithmetic Sequence:
  + Geometric Sequence:
  + Recall the variables used when writing formulas for sequences:

|  |  |
| --- | --- |
| **Variable** | **What it Represents** |
|  | The **nth term** in the sequence (the term you're trying to find) |
|  | The **first term** in the sequence |
| *n* | The **term number** (position in the sequence) |
| *d* | The **common difference** (used in arithmetic sequences) |
| *r* | The **common ratio** (used in geometric sequences) |

|  |  |
| --- | --- |
| **Example 1:** Find the function that models the geometric sequence 5, 10, 20, 40, ... . | |
| **Step 1:** Find the common ratio, *r*. | Find the common ratio by dividing the second term by the first term:  The common ratio, *r*, is 2. |
| **Step 2:** Use the explicit formula to write an explicit expression for the sequence. | * Explicit formula: * Identify the variables needed:   + First term:   + Common ratio: * Write the explicit expression: |
| **Step 3:** Rewrite the expression as a function. | The function modeling this sequence is , where *n* is the position of the term in the sequence. |

|  |  |
| --- | --- |
| **Example 2:** Find the function that models the arithmetic sequence 23, 27, 31, 35, ... . | |
| **Step 1:** Find the common difference, *d*. | Find the common difference by subtracting the first term from the second term:  The common difference, *d*, is 4. |
| **Step 2:** Use the explicit formula to write an explicit expression for the sequence. | * Explicit formula: * Identify the variables needed:   + First term:   + Common difference: * Write and **simplify** the explicit expression: |
| **Step 2:** Use the explicit formula to write an explicit expression for the sequence. | The function modeling the sequence is , where n is the position of the term in the sequence. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | The number of incoming calls received each day at a business follows the pattern 2, 5, 8, 11, 14, . . . . Identify which type of sequence that pattern represents.  Option #1: arithmetic sequence  Option #2: geometric sequence  Option #3: another type of sequence  Option #\_\_\_is correct. | 1 |
| P 2 | Given that a sequence has  and , find the third term of the sequence. Then decide which statement is true regarding the sequence.  Option #1: arithmetic sequence  Option #2: geometric sequence  Option #3: another type of sequence | 1. 0.12 2. 2 |
| P 3 | Given the arithmetic sequence , find the common difference.  The common difference of the arithmetic sequence is \_\_\_\_. | -1 |
| P 4 | A toy model manufacturing company is going to start work on a new model boat design. The company plans to produce 62 units in its first week and then increase production by 75 units for all future weeks. The total number of manufactured model boats can be modeled with the arithmetic sequence 62, 137, 212, 287, . . . . How many model boats would be manufactured in the 12th week?  The company could manufacture \_\_\_\_ model boats in the 12th week. | 887 |
| P 5 | Choose the function that models the sequence −2−2, 14, −98−98, 686, . . . .  Function #1:  Function #2:  Function #3:  Function #4: | 2 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Identify when a sequence is arithmetic. | when the difference between any two consecutive terms remains constant |
| Q 2 | Given the sequence 12, 6, 0, −6, . . . , identify the explicit formula that represents the sequence. |  |
| Q 3 | On Roy’s first birthday, his grandfather placed $100 in a bank account. On Roy’s second birthday, his grandfather placed $80 in the account and continued to place $80 each passing year. The arithmetic sequence 100, 180, 260, 340,… indicates the account balance for the first four years. Create a function that models this situation. |  |
| Q 4 | The population of an endangered species was estimated to be 240 in 2018. The population has been tracked each year and has been modeled with the geometric sequence 240, 360, 540, 810, . . . . Create a function that models this situation. |  |
| Q 5 | What is the common difference of the arithmetic sequence ? |  |

**Lesson 5 – Using a Model from a Sequence**

**Key Words:**

* **arithmetic sequence** – a sequence (such as 1, 3, 5) in which the difference between a term and its predecessor is always the same
* **geometric sequence** – a sequence (such as 1,12,14) in which the ratio of a term to its predecessor is always the same
* **sequence** – a set of numbers that follow a specific pattern or formula

**Formulas:**

* Explicit Formula to Function Form:
  + Arithmetic Sequence:
  + Geometric Sequence:

**Objective 1:** In this section, you will interpret a sequence in context using its graph and model.

*Mathematical Practice Standard: Use appropriate tools strategically.*

**Big Ideas**:

* A *sequence* can be given as a list of values, a graph, or a function that models the *sequence*.
* For example, the sequence1, 4, 9, 16, 25, 36, ... can be represented as a graph and as a function.

|  |
| --- |
| **A list of values:** 1, 4, 9, 16, 25, 36, ... |
| **Function Model:**   * A function can be used to evaluate any value of the sequence. |
| **Graph**     * The graph of a sequence shows how the sequence behaves. The x-values, or inputs, represent the term, *n*, or position of the number in the sequence. |

* A model of a *sequence* is a function that can be used to evaluate any value of the *sequence*.

|  |  |
| --- | --- |
| **Example:** The function models the sequence of the end-of-year balance of an investment account with a principal of $500 and a 5% compound annual interest rate, in which *n* is the length of the investment in years.   1. Interpret the numbers 500 and 1.05 and the variable *n* in the context of the problem. 2. Find the balance of the account after 15 years of investment. | |
| **Step 1:** Interpret the model. | * 500 represents the principal (initial deposit) * 1.05 represents the annual interest rate of 5% * *n* represents the time in years |
| **Step 2:** Find the balance after 15 years. | If *n* represents the time in years, then .  Substitute 15 for *n* and solve.      The balance of the account after 15 years of investment is $1,039.46. |

* Graphs can be used to reconstruct and interpret a *sequence*. The graph can also be used to find the function that models the *sequence*.

|  |  |
| --- | --- |
| **Example:** Carbon dating is a technique used by archaeologists to estimate the age of organic remains. When organic remains are found, archaeologists measure the level of carbon-14 in the organic remains. That information, along with the known half-life of carbon-14 being about 5,730 years, is used to calculate the age of the organic remains. Shown here is the graph of the percent of carbon-14 remaining over the number of half-lives elapsed. | |
| 1. Use the graph to rewrite the sequence of the percent of carbon-14 remaining over the number of half-lives elapsed. | The sequence can be represented by the *y*-coordinates of the graph.  The sequence is 100, 50, 25, 12.5, 6.25, 3.125, ... |
| 1. Find the function that models this graph. | * Notice that the graph models an exponential function   In the context of this problem:   * *a* is the initial value, 100. * *b* is the growth/decay factor, .   + Notice that the *y*-values of the graph are **reduced by half** after each half-life   The graph can be modeled using the function . |

**Objective 2:** In this section, you will analyze sequences and report your results using an appropriate level of precision.

*Mathematical Practice Standard: Attend to precision.*

**Big Ideas:**

* When analyzing real-world situations, it’s important to use the right level of precision.
  + Money is always reported to the nearest hundredth (2 decimal places) — even if it’s not specified.
    - Example: $3.756 should be rounded to $3.76
  + Counting objects like people, animals, or items requires whole numbers.
    - Example: You can't have 3.7 people — round to 4 people if needed.

|  |  |
| --- | --- |
| **Example:** The fish population at a local lake has been exposed to a dangerous virus. Before the outbreak, **there were 220 fish in the lake**. Now, each week, **half of the fish die from the virus**. How many fish will remain after 6 weeks? | |
| **Step 1**: Model the sequence with a list.  OR  Model the sequence with a [formula](#Bookmark2). | The lake starts with 220 fish and reduces by half each week. This can be modeled with a geometric sequence.  Model with a list:   * 220, 110, 55, 27.5, 13.75, 6.875, 3.4375...  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Weeks (term number) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | | Fish | 220 | 110 | 55 | 27.5 | 13.75 | 6.875 | 3.4375 |   Model with a formula:   * Geometric Sequence: |
| **Step 2:** Evaluate the function for x = 6. | Use the table or the formula to evaluate the amount of fish in the lake after 6 weeks (x = 6). |
| **Step 3:** Attend to precision. | Since a fish is an object, you want to report a whole number. In this case, you will round down.  There would be about 3 fish remaining in the lake after 6 weeks. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | A small town started to track its population in the year 1950. Each year that passed, the town’s population increased by 44. The function  represents the town population n years after 1950. What was the town’s population in 1950? What is the projected population in 2030? | 1. 495 2. 4,015 |
| P 2 | The cost of a large cheese pizza at a local pizzeria can be modeled with the function , where *n*is the number of toppings that can be added on. If a customer has $35 to spend, what is the maximum number of toppings that they can add to their large cheese pizza?  With $35 to spend, they can add a maximum of \_\_\_ toppings to the pizza. | 7 |
| P 3 | Use the image to answer the question.  An xy-coordinate plane's x-axis is labeled 'Elapsed Half-Lives' and ranges from 0 to 5 in unit increments. The y-axis is labeled 'Percent of Carbon-14 Remaining' and ranges from 0 to 100 in increments of 10. A solid line curve joins six plotted points.  Carbon dating is a technique used by archaeologists to estimate the age of organic remains. When organic remains are found, archaeologists measure the level of carbon 14 in the remains. That information, along with the known half-life of carbon 14 being about 5,730 years, is used to calculate the age of the organic remains.  This graph models the percentage of carbon 14 remaining over the number of half-lives elapsed. Choose the statement that describes the meaning of the coordinate point (4,6.25).  Statement #1: When 6.25 half-lives have elapsed, there is 4 percent of the carbon 14 remaining.  Statement #2: There is 6.25 percent of the carbon 14 remaining when 4 half-lives have elapsed.  Statement #3: If 6.25 percent of half-lives have elapsed, then 4 percent of the carbon 14 remains.  Statement #4: Four half-lives remain when 6.25 percent of the carbon 14 has elapsed.  Statement #\_\_\_ describes the meaning of the coordinate point (4,6.25). | 2 |
| P 4 | Use the table to answer the question.    A pendulum swinging back and forth loses momentum and distance with each oscillation. The total distance the pendulum swings on the first oscillation is 12 inches, and the distance of each consecutive oscillation is  of the previous distance. What distance will the pendulum travel on the fifth oscillation, measured to the nearest whole inch?  In the fifth oscillation, the pendulum will travel approximately \_\_\_\_ inches. | 2 |
| P 5 | There are currently 3.5 inches of snow on the ground. If the weather forecast predicts it will snow at a rate of  inch per hour over the next day, how many inches of snow would you expect to have on the ground after seven hours? Round the answer to two decimal places.  After seven hours, there will be a total of \_\_\_ inches of snow on the ground. | 5.25 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | An insect population is growing such that each new generation is 2.5 times as large as the previous generation. Suppose there were 54 insects in the first generation. The function  represents the insect population for generation *n*. Interpret the meaning of . | There were 843 insects in the 4th generation. |
| Q 2 | The sequence 3.75, 7.5, 11.25, 15, 18.75, . . . represents the total cost a customer pays at a taqueria based on the number of tacos ordered. This means that the total cost for 1 taco is $3.75, for 2 tacos is $7.50, for 3 tacos is $11.25, and so on. What is the maximum number of tacos that can be ordered if a customer has $100 to spend? | 26 tacos |
| Q 3 | The function  models the sequence of the end-of-year balance of an investment account with a principal of $275 and a 3.2 percent compound annual interest rate, in which n is the length of the investment in years. Find the balance of the account after 13 years. | $414.16 |
| Q 4 | Analyze the explicit formula of the arithmetic sequence  and report the ninth term, rounded to the nearest tenth. |  |
| Q 5 | Use the table to answer the question.    Townes bought a personal watercraft for $6,500. If it depreciates in value by 13% each year, the situation can be modeled using a geometric sequence with a common ratio of 0.87, which is started in the table. How much will the personal watercraft be worth after two years? Round the answer to the nearest whole dollar. | $4,920 |

**Lesson 6 – Creating a Model from a Dataset**

**Key Words:**

* **bivariate dataset** – a dataset that involves two variables and can be represented with a scatterplot
* **exponential function** – a function where *x* is a variable and *a* is a constant
* **interest** – a charge for borrowed money, generally a percentage of the amount borrowed
* **linear function** – an equation in the form , where *m* is the slope and *b* is the *y*-intercept
* **quadratic function** – a function in the form , where
* **regression** – a model or functional relationship between two or more variables that is often determined from data and is used most frequently to predict values of one variable when given values of the others
* **vertex form** – a way of writing a quadratic equation in which the coordinate points of the vertex are easily identifiable

**Formulas:**

* Linear Function:
  + Slope Formula:
* Quadratic Function:
  + Vertex Form:
* Exponential Function:

**Objective 1:** In this section, you will decide which type of model fits a bivariate dataset best.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* Recall that *bivariate datasets* involve two variables and can be represented with a scatter plot.
* *Linear, quadratic, and exponential functions* can be used to model data sets. When selecting a model that best fits a data set, use the following strategies:
  + If given the scatterplot, use what you know about the shapes of *linear, exponential, and quadratic functions* to determine the best match.
  + Use the context of the data to help you determine the best model. This means considering what type of change is happening in that context and connecting that type of change to a type of model.

|  |  |  |
| --- | --- | --- |
| **Linear** | **Quadratic** | **Exponential** |
| When data appears to increase or decrease in a straight line representing a constant rate of change over time. | When the y-values increase and then decrease (or decrease then increase) at the same rate, creating a parabola shape. | When the data involves a multiplier over time resulting in rapid growth or decay. |

**Objective 2:** In this section, you will create appropriate regression models given a bivariate dataset.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* [Recall](#Bookmark3) how to identify the best model for a given set of data: Linear, Quadratic, or Exponential.
* Once you identify the type of model, you can create a regression model.
  + A *regression* is a model can help you describe a dataset using an approximate equation.
  + Having a *regression* model allows you to predict future points easily by plugging in certain values.
* In a regression model, the “=” is replaced with “~” to show that these are approximate equations rather than an exact function.
  + Linear Regression Model:
  + Exponential Regression Model:
  + Quadratic Regression Model:
    - Vertex Form:
* To create a regression:
  + First, identify the best model for the given dataset.
  + Then, use the scatterplot to determine each key part of the chosen model (ex: slope, y-intercept, initial value, etc.)

|  |  |
| --- | --- |
| **Example:** Stephanie mows lawns in her neighborhood. She uses the base pay for gas and saves her tips for something fun. The graph below shows tips she’s earned based on yard size. She wants to predict how much tip she might get for a yard bigger than any she’s done. Based on the scatterplot, write the regression model to describe Stephanie’s data. | |
| **Step 1:** Select the type of model that best fits the data. | The data appears to increase at a constant rate, forming a linear pattern. The best model to use is a linear function: . |
| **Step 2:** Identify the key parts of the chosen model. (*Slope*) | For a linear model, you need to identify the slope or rate of change, *m*, and the y-intercept, *b*.  Slope (*m*):   * Recall slope formula: * Select two points on the scatter plot and apply the formula.   + (2,9) and (4,15)   + The slope is 3. |
| **Step 2:** Identify the key parts of the chosen model. (*y-intercept*) | *y-*intercept (*b*):   * This graph does not show a point on the *y*-axis. * Using the slope-intercept form of a linear function, , plug in the slope, *m*, and a point on the plot, *x* and *y*.   + and point (2,9) |
| **Step 3:** Write the linear regression model. | This data can be approximately modeled with the function . |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | When Pinocchio tells a lie, his nose grows to approximately 30% of its original size. You’ve collected data comparing the number of lies he has told and the size of his nose after each lie. Based on the context of the data, which model is the **best** fit for this data?  Option #1: linear  Option #2: quadratic  Option #3: exponential | 3 |
| P 2 | Use the image to answer the question.  Twenty points are plotted on a scatterplot titled Hours Playing with Toys. The x-axis is labeled Age, and the y-axis is labeled Hours.  The scatterplot represents data collected about how often students between the ages of 2 and 18 play with toys. Each point represents a piece of data collected from each student. For example, (8,7)(8,7) represents an 8-year-old who plays with toys for 7 hours a day. Which model is the best fit for this data?  Option #1: linear  Option #2: quadratic  Option #3: exponential | 2 |
| P 3 | Use the image to answer the question.  A scatterplot has 11 closed points plotted on a coordinate plane.  Two balls are hurled at a constant rate and a consistent angle from the top of a 35-foot-tall building. The preceding scatterplot represents the number of seconds it took for each ball to reach the ground from a certain height. Each point represents a piece of data collected from each throw. For example, (4,9) represents that after 4 seconds a ball will be 9 feet in the air. Which model is the **best** fit for this data?  Option #1: linear  Option #2: quadratic  Option #3: exponential | 2 |
| P 4 | Which form is used when a regression model is best described by a linear function?  Option #1:  Option #2: or  Option #3: | 1 |
| P 5 | Use the image to answer the question.  On a graph titled Basketball Card Values, a curve and twenty-two points are plotted in quadrant 1 of a coordinate plane. The x-axis is labeled Years, and the y-axis is labeled Dollars.  What type of function best describes the data of the regression model?  Option #1: a linear function that is in the form  Option #2: a quadratic function that is in the form  Option #3: an exponential function that is in the form | 3 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Use the image to answer the question.  On a graph titled Time Spent and Test Grades, a line and ten points are plotted in quadrant 1 of a coordinate plane. The x-axis is labeled Hours Worked, and the y-axis is labeled Grade.  The data in the graph represents the relationship between the amount of time spent studying and test grades. Each point represents a piece of data collected from students who took a test and the grade that student received. For example, (2,70) represents a student who studied for 2 hours and received a 70. Decide which model **best** fits this data. | a linear model |
| Q 2 | Use the image to answer the question.  Twenty-two points are plotted on a scatterplot titled Basketball Card Values. The x-axis is labeled Years, and the y-axis is labeled Dollars.  The data in the preceding graph represents the value of basketball cards over time. Each point represents the value of a basketball card after a certain number of years. For example, (9,50) represents a 9-year-old basketball card that is worth $50. Which model **best** fits this data? | an exponential model |
| Q 3 | Use the image to answer the question.  On a graph titled Hours Playing with Toys, a curve and twenty points are plotted in quadrant 1 of a coordinate plane. The x-axis is labeled Age, and the y-axis is labeled Hours.  Jorge needs to create an appropriate regression model given the bivariate dataset. Which of the following should he create? |  |
| Q 4 | Use the image to answer the question.  On a graph titled Time Spent and Test Grades, a line and ten points are plotted in quadrant 1 of a coordinate plane. The x-axis is labeled Hours Worked, and the y-axis is labeled Grade.  Which of the following is an appropriate regression model given the bivariate dataset? |  |
| Q 5 | Use the image and table to answer the question.  An illustration shows a scatterplot titled Number of Laps the Swim Team Completes Per Week. The x-axis shows weeks ranging from 1 through 10 in increments of one. The y-axis shows number of laps and ranges from 100 to 1,000 in 100 unit increments.    Which of the following is an appropriate regression model, rounded to the nearest tenth, given the bivariate dataset? |  |

**Lesson 7 – Using a Model from a Dataset**

**Key Words:**

* **bivariate analysis** – used to determine if there is a relationship between two values
* **bivariate data** – the data that contains two quantitative variables
* **correlation** – the relationship between variables
* **correlation coefficient** – a number (usually represented with the variable *r*) in the range of that indicates how accurately an equation represents the data being examined
* **line of best fit** – a straight line in which the distance between it and the data is minimized
* **linear relationship** – a pattern created when the relationship between the independent and dependent variables remains constant
* **negative association** – the relationship between points on a scatterplot where the *x*-values tend to increase as *y*-values decrease
* **no association** – the relationship between points on a scatterplot that show no obvious pattern to the data
* **nonlinear relationship** – a pattern created when the relationship between the independent and dependent variables does not remain constant
* **outlier** – a data point that is significantly different than all of the other data points
* **positive association** – the relationship between points on a scatterplot where the *x*-values tend to increase as *y*-values increase
* **regression** – a model or functional relationship between two or more variables that is often determined from data and is used most frequently to predict values of one variable when given values of the others
* **scatterplot** – a two-dimensional graph in rectangular coordinates consisting of points whose coordinates represent values of two variables under study

**Objective 1:** In this section, you will interpret a bivariate dataset in context using its scatterplot and regression model.

*Mathematical Practice Standard: Use appropriate tools strategically.*

**Big Ideas**:

* *Bivariate* data can be interpreted using its *scatterplot* and [regression model](#Bookmark4).
* Recall the key features used to interpret and describe datasets represented on *scatterplots*; relationship, association, and strength.

|  |  |
| --- | --- |
| **Relationship** | |
| **Association** |  |
| **Relationship**  **(Linear vs Nonlinear)** | **Linear:** Forms a line that increases or decreases from left to right when graphed.    **Nonlinear:** Graph shows a relationship, but not with a straight line. |
| **Strength** | **Strong:** Points close together forming a tighter pattern.  **Weak:** Points are spread out from the pattern. |
| **Outliers** | A data point that is significantly different than other data points. |

|  |  |
| --- | --- |
| **Example:** Interpret the bivariate data by using its scatterplot and regression model. Describe the association, relationship, strength, and outliers. | |
|  | * The association is positive. The linear relationship increases as you move from left to right. * The relationship is linear because the points closely resemble a straight line. * The strength is strong because the points closely follow a straight line. * There is an outlier at approximately . This point is different from all the other points in that it lies outside the pattern of the other points. |
|  | * There is no association. * The relationship is nonlinear because the points do not resemble a straight line. * The strength is strong because the points closely follow a curved line. * There are no outliers. |

**Objective 2:** In this section, you will report bivariate data analysis results with an appropriate level of precision.

*Mathematical Practice Standard: Attend to precision.*

**Big Ideas:**

* There are three ways to perform a *bivariate analysis* to determine if there is a relationship between two values.
  + scatterplots
  + regression analysis
  + correlation coefficient (linear only)

|  |  |
| --- | --- |
| **Example:** Use the data to report a bivariate data analysis. Examine the scatterplot, regression, and the correlation coefficient. Round to two decimal places. | |
| **Step 1:** Create a scatterplot using the given data. |  |
| **Step 2:** Describe the association, relationship, strength, and outliers. | * The scatterplot forms a line, indicating the relationship is linear. * The relationship decreases from left to right, indicating the association is negative. * The strength is strong because the points closely follow a straight line. * There are no outliers. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Use the image to answer the question.  On a graph titled Hours Playing with Toys, a curve and twenty points are plotted in quadrant 1 of a coordinate plane. The x-axis is labeled Age, and the y-axis is labeled Hours.  Does this dataset represent a strong or weak relationship?  Option #1: strong relationship  Option #2: weak relationship | 1 |
| P 2 | Use the image to answer the following question.  On a graph titled Sweater Sales by Temperature, a line and seventeen points are plotted in quadrant 1 of a coordinate plane. The x-axis is labeled Temperature in degrees Fahrenheit, and the y-axis is labeled Sales.  Which option **best** describes the relationship between sweater sales and temperature?  Option #1: Sweater sales increase as the temperature increases.  Option #2: Sweater sales decrease as the temperature increases.  Option #3: Sweater sales remain the same as the temperature increases. | 2 |
| P 3 | Use the image to answer the question.  Twenty-one points are plotted on a coordinate plane. The x-axis ranges from 0 to 12 in increments of 2. The y-axis ranges from 0 to 12 in increments of 2.  What type of association is represented by this scatterplot?  Option #1: a positive association  Option #2: a negative association  Option #3: no association | 3 |
| P 4 | Use the table to answer the question.    Use the data to report a bivariate data analysis. The correlation coefficient is . Determine whether the association is positive or negative and whether it is strong or weak.  Option #1: strong positive association  Option #2: weak positive association  Option #3: strong negative association  Option #4: weak negative association | 1 |
| P 5 | Use the table to answer the question.    Use the data to report a bivariate data analysis. The correlation coefficient is . Which of the following statements must be true?  Option #1: Since r is close to 1, there is a strong positive linear association between the dollars spent and the number of gallons of gas obtained.  Option #2: Since r is not close to 1, there is no association between the dollars spent and the number of gallons of gas obtained.  Option #3: Since r is close to 1, there is a weak positive linear association between the dollars spent and the number of gallons of gas obtained.  Option #4: Since r is close to −1, there is a strong negative linear association between the dollars spent and the number of gallons of gas obtained. | 1 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Use the image to answer the question.  A solid line and fifteen data points are plotted on a scatterplot titled Sweater Sales by Temperature. The x-axis is labeled Temperature in Degrees Fahrenheit, and the y-axis is labeled Quantity of Sweaters Sold.  Which of the following interprets the **most likely** cause of the outlier in this scatterplot? | There is a one-day sale on sweaters on a warm summer day. |
| Q 2 | Use the image to answer the question.  On a graph titled Basketball Card Values, a curve and twenty-two points are plotted in quadrant 1 of a coordinate plane. The x-axis is labeled Years, and the y-axis is labeled Dollars.  Which of the following **best**describes the relationship this dataset represents? | nonlinear |
| Q 3 | Use the image to answer the question.  Twenty-one points are plotted on a coordinate plane. The x-axis ranges from 0 to 12 in increments of 2. The y-axis ranges from 0 to 12 in increments of 2.  Which of the following **best** describes whether the relationship is strong or weak, and why it is that way? | The relationship is weak because the points do not model a straight line or curved pattern. |
| Q 4 | Use the table to answer the question.    Use the data to report a bivariate data analysis. The correlation coefficient is . Which of the following **best** reports the relationship between the number of absences and the final grade? | As the number of absences increase, the final grade decreases. |
| Q 5 | Use the table to answer the question.    Use the data to report a bivariate data analysis. The correlation coefficient is . Which of the following statements must be true? | The more games rented, the more dollars spent. |

**Lesson 8 – Creating a Model from a Verbal Description**

**Key Words:**

* **exponential function** – a function where *x* is a variable and *a* is a constant
* **linear function** – a function that represents a straight line on the coordinate plane
* **quadratic equation** – a function that represents a parabola on the coordinate plane

**Formulas:**

* Linear Function:
* Exponential Function:
* Quadratic Function:
  + Vertex Form:

**Objective 1:** In this section, you will decide if a linear, quadratic, or exponential model will best represent a verbal description of a context.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* *Linear, quadratic, and exponential* functions have specific shapes and features. Real-world contexts can be modeled using these functions.
  + Recall how the graphs of specific models behave. This is helpful to refer to when you only have a verbal description of data.

|  |  |  |
| --- | --- | --- |
| **Positive Linear** | Models data that increases at a constant rate. |  |
| **Negative Linear** | Models data that decreases at a constant rate. |  |
| **Positive Exponential (Growth)** | Models data that increases rapidly. |  |
| **Negative Exponential (Decay)** | Models data that decreases rapidly. |  |
| **Positive Quadratic** | Models data that decreases to a minimum and then increases. |  |
| **Negative Quadratic** | Models data that increases to a maximum and then decreases. |  |

|  |  |
| --- | --- |
| **Example:** Decide if a linear, quadratic, or exponential model will best represent the following situations | |
| **1.)** Jaelyn wants to know how the height of a football as a placekicker attempts a 40-foot field goal. What type of graph will show the distance the football travels once kicked? | Answer: A negative quadratic model best represents this context.  Explanation: After the football is kicked, it travels upwards and reaches a maximum height, then it returns to the ground, creating a parabolic path. |
| 2.) A taxi driver charges you a $5 minimum, plus $0.50 per mile to your destination. | Answer: a positive linear function  Explanation: For each mile traveled, the cost increases at a constant rate of $0.50. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | A golfer wants to model the motion of her golf shot on a graph. She sets the x-axis as time and the y-axis as elevation. What kind of function **best** represents this scenario?  Option #1: a linear function  Option #2: a quadratic function  Option #3: an exponential function | 2 |
| P 2 | In epidemiology, the study of infectious diseases, scientists measure how fast a disease spreads using Ro, which is also known as the coefficient of transmission. An Ro score of 1 means that each sick person will infect one other person. An Ro score of 2.5 means that each sick person infects an average of 2.5 other people. Which function type represents the spread of a disease with an Ro of 2?  Option #1: a linear function  Option #2: a quadratic function  Option #3: an exponential function | 3 |
| P 3 | Which type of slope is **best** to use to create a model for measuring the elevation versus time of a train traveling up a mountain?  Option #1: zero slope  Option #2: positive slope  Option #3: negative slope | 2 |
| P 4 | Which type of slope is **best** to use to create a model for measuring the speed versus the time of a car that is approaching a red light?  Option #1: zero slope  Option #2: positive slope  Option #3: negative slope | 3 |
| P 5 | Which type of slope is **best** to use to create a model for determining the amount of money in the bank each month for a person who is not making monthly contributions and is still paying bills from the account?  Option #1: zero slope  Option #2: positive slope  Option #3: negative slope | 3 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Consider the scenario: A job raking leaves pays $10 to get started, then $15 for every hour of work. Decide which type of function **best** represents how much money the leaf-raker make at a worksite. | a positive linear function |
| Q 2 | Quavarius sets up an investment account that guarantees a 5% interest compounded monthly. He deposits $100 into his account each month. Which type of function **best** models the amount of money Quavarius has in his account? | a positive exponential function |
| Q 3 | Which scenario would create a graph with a positive slope? | the amount of money that someone makes at a job over the course of a week |
| Q 4 | Which graph would **best** model the relationship between hours studied and student performance on a 15-question test? | Sixteen data points are plotted on a scatterplot. |
| Q 5 | Which graph would **best** model the relationship between a student’s shoe size and their grade in math class? | Twenty-one points are plotted on a coordinate plane. The x-axis ranges from 0 to 12 in increments of 2. The y-axis ranges from 0 to 12 in increments of 2. |

**Lesson 9 – Using a Model from a Verbal Description**

**Key Words:**

* **exponential function** – a function where *x* is a variable and *a* is a constant
* **linear function** – an equation in the form , in which *m* is the slope and *b* is the *y*-intercept
* **quadratic equation** – any equation containing one term in which the unknown is squared and no term that is raised to a higher power

**Formulas:**

* Linear Function:
* Exponential Function:
* Quadratic Function:
  + Vertex Form:

**Objective 1:** In this section, you will interpret a verbal description of a context using its function and graph.

*Mathematical Practice Standard: Use appropriate tools strategically.*

**Big Ideas**:

* [Recall](#Bookmark5) how to model different types of functions from a story or real-world context.
* You can also reverse this and create a story from a given model.
* Review the shapes of each type of slope and examples of how they can model real-world situations:

|  |
| --- |
| **Slope** |
| Any situation where there is an increase.  Example: going up stairs |
| Any situation where there is a decrease.  Example: spending money |
| Any situation where there is no change.  Example: not earning or spending any money |

|  |  |
| --- | --- |
| **Example 1:** Use the diagram to create a story that fits the population-versus-time data displayed. | Sample Answer: The population grows at a constant linear rate for some time, then it decreases at a constant linear rate for a while. The population then grows at a constant linear rate that is slower than the original rate for the rest of the time displayed on the graph. |
| **Example 2:** Use the provided equation to create a story that fits Jose’s savings account when represents the number of weeks. | Sample answer: Jose’s savings account started with $125 and he deposits $15 every week into the account. |

**Objective 2:** In this section, you will analyze a verbal description of a context using its function and graph with an appropriate level of precision and report the results.

*Mathematical Practice Standard: Attend to precision.*

**Big Ideas:**

* Recall, that when modeling a problem, the following rules can help you recognize what type of function is needed.
  + Linear
    - The problem is about repeated addition or subtraction of a constant value
  + Exponential:
    - The problem is about repeated multiplication or division of a constant value
    - The problem involves compound interest
    - The problem is about population growth or decay
  + Quadratic:
    - The problem involves a free-falling object
    - The problem involves determining area
* Recall that you can use a graph, a table, or an equation to solve real-world problems.
  + First, determine the behavior of the situation and what type of function would best fit.
  + Next, develop an equation to fit the specifics of the situation.
  + Then, model the equation using a table and/or graph.

|  |  |
| --- | --- |
| **Example:** Ron wants to purchase a VR gaming system that is currently on sale for $500. Ron does not have the money saved. His older sister Ginny has offered to lend Ron the Graphing Calculator money as long as he comes up with a plan to repay her. After examining his income and expenses, Ron has decided he can repay Ginny $25 a week. How many weeks will it take Ron to repay Ginny the $500 loan? | |
| **Step 1:** Determine the behavior of the situation and what type of function would best fit. | This situation shows a decreasing function because the loan gets smaller over time. Since the amount goes down by the same $25 each week, it can be modeled with a **linear equation**. |
| **Step 2:** Develop an equation to fit the specifics of the situation. | * Identify the variables:   + Let x = number of weeks   + Let y = amount of loan Ron still owes * Determine the starting amount (y-intercept):   + This is how much Ron owed at the beginning (before any payments).   + *b =* 500 * Find the slope (rate of change):   + Since Ron pays the same amount each week, the slope is negative (the loan is decreasing).   + *m =* -25 * Write the equation:   + Use the form of a linear function:   + The situation can be represented with the function |
| **Step 3:** Model the equation using a table and/or graph. | Table:    Graph: |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Use the image to answer the question.  A graph shows a step line plotted.  According to the graph, how many times did Lionel incur an expense?  Lionel incurred an expense \_\_\_ time(s). | 2 |
| P 2 | Use the image to answer the question.  4 lines are plotted on a coordinate plane.  The graph models the cost of hiring an entertainer for an event. What is the cost of hiring the entertainer for 3.5 hours?  The cost of hiring the entertainer for 3.5 hours is $\_\_\_\_. | 100 |
| P 3 | Use the image to answer the question.  A coordinate plane's axes range from 0 to 14, both by 2-unit increments. The x-axis is labeled 'Time' with minute units, and the y-axis is labeled 'Distance' with mile units. A line is plotted with 3 different sections.  The graph models the distance a car travels over time. At what minute did the car stop moving? At what minute did the car start moving again? | 1. 5 2. 10 |
| P 4 | Which option contains examples of ways to attend to precision in mathematics?  Option #1: Make accurate calculations, provide answers with appropriate units, and label the axis on graphs.  Option #2: Use only a calculator, complete problems quickly by writing down minimal work, and always round to a whole value.  Option #\_\_\_\_ provides a list of ways to attend to precision. | 1 |
| P 5 | The path of a soccer ball can be modeled by the equation , where  is the height reached by the ball, in feet, and *x* is the horizontal distance the ball has traveled, in feet. What is the horizontal distance the ball will have traveled when it hits the ground? Use a graph or table if necessary.  The ball will have traveled a horizontal distance of \_\_\_ feet when it hits the ground. | 20 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Use the image to answer the question.  A coordinate plane's axes range from 0 to 14, both by 2-unit increments. The x-axis is labeled 'Time' with minute units, and the y-axis is labeled 'Distance' with mile units. A line is plotted with 3 different sections.  Which of the following descriptions **best** interprets the graph? | A car traveled at a speed of 1 mile per minute. After 5 minutes, the car stopped to wait for a train to cross, which took 5 minutes. Then the car continued to its destination. |
| Q 2 | Use the image to answer the question.  A coordinate plane with solid, dotted and dotted dashed lines plotted. Both axes ranges from negative 5 to 5 in unit increment.  If the graph models the movement of an object, which of the following x-values indicates no movement? |  |
| Q 3 | Analyze the answer choices and select the scenario in which an exponential function should be used to represent the problem. | The problem is about the repeated multiplication or division of a constant value. |
| Q 4 | Use the image to answer the question.  A concave curve is plotted in quadrant 1 of a coordinate plane. Eleven unlabeled points are plotted on the curve.  Carla is watching a professional baseball game on television and sees a batter hit a home run over the fence. A graphic overlay comes up during the replay that shows how high and far the ball traveled in feet. Carla recognizes the shape to be a quadratic and wants to write an equation to represent the function. Which of the following functions accurately models the graph? | The height reached by the ball, , is given by the equation , where *x* represents the horizontal distance. |
| Q 5 | Use the table to answer the question.    The table displays success rates for two different techniques used to memorize multiplication facts. The table shows how many multiplication facts a child memorized after the given number of days using either Method A or Method B. What type of function should be used to model each method? | Method A should be modeled with a linear equation, and Method B should be modeled with a quadratic equation. |