Transformations of Functions

**Formula Sheet**

|  |  |  |
| --- | --- | --- |
| **Name** | **Definition** | **Formula** |
| Parent Function | A parent function is a function without transformations and is the most basic form of a function. |  |
| Vertical Reflection | A vertical reflection is a reflection over the *x-*axis, or a flip over the *x-*axis. | Equation Rule: $$y=f\left(x\right) to y=−f\left(x\right)$$Coordinate Rule: $$\left(x,y\right) to \left(x,−y\right)$$ |
| Horizontal Reflection | A horizontal reflection is a reflection over the *y*-axis, or a flip over the *y*-axis. | Equation Rule: $$y=f\left(x\right) to y=f\left(−x\right)$$Coordinate Rule: $$\left(x,y\right) to \left(−x,y\right)$$ |
| Reflection about the Origin | When a reflection occurs over both the *x*- and *y-*axes, it is called a reflection about the origin, which has the coordinates (0, 0).  | Equation Rule: $$y=f\left(x\right) to y=−f\left(−x\right)$$Coordinate Rule: $$\left(x,y\right) to \left(−x,−y\right)$$ |
| Vertical Shift | A vertical shift is a translation that shifts the graph of a function vertically, up or down. | Equation Rule: $$y=f\left(x\right) to y=f\left(x\right)\pm k$$Coordinate Rule: $$\left(x,y\right) \rightarrow  \left(x, y\pm k\right)$$

|  |  |  |
| --- | --- | --- |
|  | ***Shift Up*** | ***Shift Down*** |
| ***Equation*** | $$ y=f\left(x\right)+k$$ | $$ y=f\left(x\right)−k$$ |
| ***Coordinate*** | $$\left(x,y\right) \rightarrow  \left(x, y+k\right)$$ | $$\left(x,y\right) \rightarrow  \left(x, y−k\right)$$ |

 |
| Horizontal Shift | A horizontal shift is a translation that shifts the graph of a function horizontally, left or right. | Rule: $$y=f\left(x\right) to y=f\left(x\pm k\right)$$

|  |  |  |
| --- | --- | --- |
|  | ***Shift Left*** | ***Shift Right*** |
| ***Equation*** | $$y=f\left(x+k\right)$$ | $$y=f\left(x−k\right)$$ |

 |
| Vertical Dilation | A vertical dilation stretches or compresses the graph of a function vertically.  | Equation Rule: $$y=f\left(x\right) to y=kf\left(x\right)$$Coordinate Rule: $$\left(x,y\right) to \left(x,ky\right)$$* $$k>1 $$; vertical stretch by a factor of *k* units.
* $$0<k<1 $$; vertical compression by a factor of *k* units.
 |
| Horizontal Dilation | A horizontal dilation stretches or compresses the graph of a function horizontally. | Rule: $$y=f\left(x\right) to y=f\left(kx\right)$$* $$0<k<1 $$; horizontal stretch by a factor of *k* units.
* $$k>1 $$; horizontal compression by a factor of *k* units.
 |
| Calculate *k* factor | The exact value of k may be determined by comparing two points on the graph.  | Compare a point from the original graph and the transformed graph with the same *y-*value.$$k=\frac{x−value of original graph}{x−value of transformed graph}$$ |
| Even Function | Even functions return the same expression for both *x* and *–x*. The output, or *y-*value, will be the same if the *x*-value is positive or negative. | $$f\left(x\right)$$ is an even function when $$f\left(x\right)=f\left(−x\right)$$* In a table, the $$f\left(x\right)$$ and $$f\left(−x\right)$$ values are identical
* On a graph, the reflection is symmetric about the *y*-axis
 |
| Odd Functions | Odd functions are symmetric when reflected about the origin.  | $$f\left(x\right)$$ is an odd function when $$f\left(−x\right)=−f\left(x\right)$$ |