Algebra 1

**Transformations of Functions**

**Unit Summary:** In this unit, you will explore how to apply transformations to parent functions. These transformations include translations, reflections, stretches, and compressions. You will learn how to graph and write equations that represent these transformations, as well as identify, graph, and write equations for multiple transformations combined.

**Lesson 2 – Parent Functions**

**Key Words:**

* **dilation** – a transformation that stretches or compresses the graph of a function horizontally or vertically
* **family of functions** – the functions created from transformations of a parent function
* **horizontal compression** – a dilation that laterally compresses the graph of a function; occurs when the x-value of a function is multiplied by a constant, k, whose value is greater than 1
* **horizontal reflection** – a reflection of the graph of a function over the y-axis
* **horizontal shift** – a translation that shifts the graph of a function horizontally
* **horizontal stretch** – a dilation that laterally stretches the graph of a function; occurs when the x-value of a function is multiplied by a constant, k, whose value is between 0 and 1
* **parent function** – a function without transformations
* **reflection** – a transformation that reflects the graph of a function over a horizontal or vertical line
* **slope** – the steepness of a line, found by dividing the change in the y-value by the change in the x-value
* **transformation** – the manipulation of an equation or graph from the parent function to change its position, size, or shape
* **translation** – a transformation that shifts the graph of a function vertically or horizontally
* **vertical compression** – a dilation that compresses the graph of a function vertically
* **vertical reflection** – a reflection of the graph of a function over the x-axis
* **vertical shift** – a translation that shifts the graph of a function vertically
* **vertical stretch** – a dilation that stretches the graph of a function vertically

**Objective 1:** In this section, you willuse graphs, equations, and tables to informally show that every linear function is an altered version of .

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* Recall that *slope* affects the steepness and direction of a liner function.
  + The higher the *slope*, the steeper the line.
  + Positive *slope*: the line will go up from left to right
  + Negative *slope*: the line will go down from left to right

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| **Slope Review** | |
| * Line A: * Line B: * Line C: * Line D: |  |

* Recall that linear functions demonstrate the relationship where *x* is the input and *f(x)* is the output.
  + Determine the outputs of a function by inserting the *x*-value into the function and calculating.
* All linear functions are adapted from the original, or ***parent function***, , through the use of mathematical operations.

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| **Example:** Describe the differences between the two graphed functions. | |
| * Linear Function 1: * Linear Function 2:   Both linear functions have the same positive slope. They are going in the same direction with the same steepness.  Linear function 2 has been moved up six units on the *y-*axis. |  |

**Objective 2:** In this section, you will show how a single simple parent function can be transformed to create a whole family of functions.

*Mathematical Practice Standard: Construct viable arguments and critique the reasoning of others.*

**Big Ideas:**

* Out of one *parent function* you can create a whole *family of functions* by *transformation*.
* A *transformation* is manipulating an equation or graph from the *parent function* to change its position, size, or shape.
  + *Transformations* include reflections, rotations, translations, and dilations.
* A *parent function* is a function **without** *transformations* and is the most basic form of a function.

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| **Parent Functions** |
| Linear Function: |
| Quadratic Function: |
| Cubed Function: |
| Absolute Value Function: |
| Square Root Function: |
| Cube Root Function: |

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| **Types of Transformations** |
| **Vertical Reflections:**   * A reflection over the *x-*axis (flipped over the *x*-axis) * It changes the equation to * Example: The parent function is transformed to by a reflection over the *x-*axis. |
| **Horizontal Reflections:**   * A reflection over the *y-*axis (flipped over the *y*-axis) * It changes the equation to * Example: The parent function is transformed to by a reflection over the *y-*axis. |
| **Vertical Shifts:**   * Translation that shifts the graph of a function vertically * It changes the equation from to * The graph is up or down from the parent function by *k* units   + : vertical shift up   + : vertical shift down * Example: The parent function is transformed to by a vertical shift **up** 3 units. |
| **Horizontal Shifts:**   * Translation that shifts the graph of a function horizontally * It changes the equation from to * The graph is moved left or right from the parent function by *k* units   + : horizontal shift right   + : horizontal shift left * The parent function is transformed to by a horizontal shift **left** two units. |
| **Vertical Dilations:**   * Dilation that **stretches** or **compresses** the graph of a function vertically * It changes the equation from to   + : stretch   + : compression * Example: The parent function is transformed to by a vertical compression by a factor of . |
| **Horizontal Dilations:**   * Dilation that **stretches** or **compresses** the graph of a function horizontally * It changes the equation from to   + : stretch   + : compression * Example: The parent function is transformed to by a horizontal stretch by a factor of . |

**Objective 3:** In this section, you will describe transformations from parent functions.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Big Ideas:**

* When describing functions in relation to their *parent functions*, consider the following steps:
  + 1. Define the general shape of the function. This will help you identify the *parent function*.
  + 2. List similarities and differences between the new function and its *parent function*.
* When comparing and describing the graph of a *parent function* and a *transformed* version, it’s useful to determine whether:
  + The size has changed – narrow or wider?
  + The shape has been reflected to create a mirror image
  + The shape has changed position – moved up, down, left, or right

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| **Example**: Consider the following graph. How does Function B differ from its parent function, Function A?    The U-shape of the graph of Function shows that it is transformed from the quadratic parent function, Function A. The graph of Function B that is displayed is wider and has slid to the right of the origin. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Based on the tables, describe the difference from Linear Function 1 to Linear Function 2.  Linear Function 1    Linear Function 2    The outputs from Linear Function 2 are increased by \_\_\_\_\_ units. | 5 |
| P 2 | Use the image to answer the question.  Two lines marked line L and line K are drawn on a coordinate plane. Both axes range from negative 10 to 10 in unit increments.  Describe the difference between line K and line L based on the pictures.  Line K has been moved up \_\_\_ units on the y-axis to line L. | 8 |
| P 3 | Which function is a transformation of the parent function ?  Option #1:  Option #2:  Option #3: | 2 |
| P 4 | Use the image to answer the question.  A coordinate plane's axes range from negative 5 to 5, both by 1-unit increments. 2 functions are plotted, 1 solid and 1 dotted. The solid function is labeled 'Function upper A' and the dotted function is labeled 'Function upper B.'  Describe the transformation from Function A to Function B. Enter the value of the transformation.  The graph is narrower and shifted to the right by \_\_\_ units. | 2 |
| P 5 | Use the image to answer the question.  A coordinate plane's axes range from negative 10 to 10, both by 1-unit increments. 2 lines are plotted, 1 solid and 1 dotted. The dotted line is labeled 'Function upper B' and the solid line is labeled 'Function upper A.'  Describe the transformation from Function A to Function B. Select the option number that corresponds to the correct direction of the translation after the reflection. Then enter the number of units that the function was translated.  Option 1: down  Option 2: up  The function is reflected over the *y*-axis and then shifted \_\_\_ by \_\_\_ units. | 1; 2 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Use Equation A and describe the differences to show the change from  to the equation A.  Equation A: | The slope increases by a factor of 3 and the graph of the function moves down the *y*-axis 4 units. |
| Q 2 | Use the image to answer the question.  Two lines marked line X and line W are drawn on a coordinate plane. Both axes range from negative 10 to 10 in unit increments.  How does the graph show the change from Line W to Line X? | There is a slope change from positive to negative and by a factor of 3. |
| Q 3 | Which of the following statements shows that parent functions can create all other functions in a function family? | Parent functions can be translated, reflected, and/or dilated to create all functions within a family. |
| Q 4 | Use the image to answer the question.  A coordinate plane's axes range from negative 5 to 5, both by 1-unit increments. 2 functions are plotted, 1 solid and 1 dotted. The solid function is labeled 'Function upper A' and the dotted function is labeled 'Function upper B.'  Describe the transformations from Function A to Function B. | wider and shifted right 2 units |
| Q 5 | A quadratic function on a graph has the vertex at the point (−3,−2). Which of the following transformations translates the vertex to the point (0,0)? | shift right 3 units and up 2 units |

**Lesson 3 – Reflections About the x-axis**

**Key Words:**

* **parent function** – a function without transformations
* **reflection** – a transformation that reflects the graph of a function over a horizontal or vertical line
* **transformation** – a manipulation of a function so that its graph is translated, reflected, rotated, or dilated
* **vertical reflection** – a reflection of the graph of a function over the x-axis

**Formulas:**

* Vertical Reflection Equation:
* Vertical Reflection Coordinates:

**Objective 1:** In this section, you will show using tables and graphs how results in the function being reflected over the x-axis.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* [Recall](#Bookmark2) that a *vertical reflection* is a reflection over the *x-*axis, or a flip over the *x-*axis.
* It is the transformation that occurs to a function when a negative sign is applied to the function.
  + Equation:
* When a *vertical reflection* occurs, the *x-*values stay the same as the parent function, but the *y-*values are opposite of the parent function.
  + Coordinate Points:
  + The negative sign before the *y-*coordinate doesn’t mean the number is negative; it indicates a sign change. A negative value becomes positive, and a positive value becomes negative.

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| **Example:** What type of transformation occurs with when ? | |
| **Step 1:** Create a table for each function using the same inputs and evaluate each function. | |  |  | | --- | --- | | ***x*** |  | | 1 |  | | 4 |  | | 9 |  | | 16 |  |  |  |  | | --- | --- | | ***x*** |  | | 1 |  | | 4 |  | | 9 |  | | 16 |  | |
| **Step 2:** Compare the outputs for and . | Although the inputs, *x-*values, are the same for both functions, the outputs, *y-*values, are opposites of each other. Therefore, this must be a reflection over the *x-*axis. |

**Objective 2:** In this section, you will describe the transformation from to function graphs of the form .

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* Two things you should notice when looking at the graph of a *parent function* (original function) and its *vertical reflection*:
  + The new function is created by flipping the original function over the x-axis (flipping the image upside down)
  + All of the *x*-values stay the same, while all of the *y*-values become their opposite (multiplying all of the outputs by –1)

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| **Example:** Identify which graph is a vertical reflection of ? | |
| **Step 1:** Look at the location. | **Graph B** is the vertical reflection of because the transformed function is flipped over the *x*-axis and is mirroring the original function. |
| **Step 2:** Look at the points. | The *x-*coordinates change to their opposites while the *y-*coordinates stay the same.   |  |  | | --- | --- | |  | *Graph B* | | (0, 4) | (0, -4) | | (-4, 4) | (-4, -4) | |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Create a table for the reflection over the x-axis of the function . | -1; 0; -1 |
| P 2 | Based on the following tables, which of the following options is a true description of the graphs of f(x) and g(x)?    Option #1: One graph is a reflection of the other over the x-axis.  Option #2: One graph is a reflection of the other over the y-axis.  Option #3: One graph is a reflection of the other about the origin. | 1 |
| P 3 | If the point (3,4) is on the graph of , what point must be on the graph of ?  Option #1: (−3,4)  Option #2: (−3,−4)  Option #3: (3,−4)  The point in option \_\_\_ must be on the graph of | 3 |
| P 4 | Use the image to answer the question.  Two parabolas and two inverted parabolas, all with arrows on both ends, are drawn on a coordinate plane with the x-axis from negative 6 to 6 and the y-axis from negative 6 to 6, both in increments of 1.  Given the graph of , which of the options is the graph of ?  The graph of is option \_\_\_\_. | 2 |
| P 5 | Use the image to answer the question.  Four concave curves are drawn on a coordinate plane with the x-axis from negative 6 to 6 and the y-axis from negative 4 to 4, both in increments of 2.  Given the graph of , which of the following options is the graph of ? Enter the option number of the correct answer.  Option #1: Graph B  Option #2: Graph C  Option #3: Graph D | 3 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | If the function  is reflected over the x-axis, then how does the reflection affect the point (2,25)? | In the reflection, the point (2,25) becomes (2,−25). |
| Q 2 | Use the image to answer the question.  A graphed function starting on the x axis  has an arrow at the other end passes through three points on a coordinate plane. The x-axis ranges from negative 9 to 9 in unit increments and the y-axis ranges from negative 11 to 11 in unit increments.  Which table shows that  results in a reflection over the x-axis? |  |
| Q 3 | Choose the correct statement to describe a true relationship between any point on f(x) and −f(x). | If (x,y) is on the graph of f(x), then (x,−y) is on the graph of −f(x). |
| Q 4 | Use the image to answer the question.  Four lines with bidirectional arrows are plotted and labeled on a coordinate plane. The equation f left parenthesis x right parenthesis equals x plus 1 appears at the top of the graph in quadrant 1.  Given the graph of , which of the options is the graph of ? | Option 2 |
| Q 5 | Use the image to answer the question.  Five labeled sinusoidal waves are plotted on a coordinate plane.  Given the graph of , which of the options is the graph of ? | The graph of is option 1. |

**Lesson 4 – Reflections About the y-axis**

**Key Words:**

* **horizontal reflection** – a reflection of the graph of a function over the y-axis
* **reflection** – a transformation that reflects the graph of a function over a horizontal or vertical line

**Formulas:**

* Horizontal Reflection Equation:
* Horizontal Reflection Coordinates:

**Objective 1:** In this section, you will use tables and graphs to show how results in the function being reflected over the y-axis.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* [Recall](#Bookmark2) that a *horizontal reflection* is a *reflection* over the *y-*axis, or a flip over the *y-*axis.
* It is the transformation that occurs to a function when a negative sign is applied to the *x-*value in the function.
  + Equation:
* When a *horizontal reflection* occurs, the *y-*values stay the same as the parent function, but the *x-*values are opposite of the parent function.
  + Coordinate Points:
  + The negative sign before the *x-*coordinate doesn’t mean the number is negative; it indicates a sign change. A negative value becomes positive, and a positive value becomes negative.

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| **Example:** Given the following table for , create a table for the function . | |
| **Step 1:** Identify the type of transformation that is occurring. | The function is a horizontal reflection because it follows the equation change: |
| **Step 2:** Create a table to show the new coordinates of the reflected function. | * Each input value (*x-*value) will be the **opposite** of the original input value. * The output values (y-values) will remain the same.  |  |  |  | | --- | --- | --- | | ***x*** | ***h(x)*** | **Coordinates** | | -2 | 1 | (-2, 1) | | -4 | 3 | (-4, 3) | | -6 | 7 | (-6, 7) | | -8 | 11 | (-8, 11) |   The horizontal flips all points in the first quadrant horizontally to the second quadrant. |

**Objective 2:** In this section, you will describe the transformation from  to function graphs of the form .

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* When looking at the graph of a *parent function* (original function) and its *horizontal reflection* you will notice these key takeaways:
  + The new function is created by flipping the original function over the *y-*axis. The reflected function is the same distance from the *y-*axis, only it is in the opposite direction of the original function.
  + All of the *y*-values stay the same, while all of the *x*-values become their opposite (multiplying all of the inputs by –1)

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| **Example:** Identify which graph is a horizontal reflection of ? | |
| **Step 1:** Look at the location. | **Graph A** is the horizontal reflection of . The transformed function is reflected over the *y-*axis and now mirrors the original function. |
| **Step 2:** Look at the points. | The *x-*coordinates have changed to their opposite while the *y*-coordinates have stayed the same.   |  |  | | --- | --- | |  | *Graph A* | | (-4, -4) | (4, -4) | | (1, 2) | (-1, 2) | |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | If the function is , complete the table for its horizontal reflection. | 0; -1; -8 |
| P 2 | Use the image to answer the question.  An parabola is drawn on a coordinate plane. Both axes range from negative 5 to 5 in one-unit increments.  Given the graph of the function f(x), complete the following table of the points for the horizontally reflected function f(−x). | -2; 0; -2 |
| P 3 | Graph the function  and the function of its horizontal reflection. What is the position of the point (−4,9) in the reflected function?  The point (−4,9) reflects to (\_\_, \_\_\_) in . | 4, 9 |
| P 4 | If the point (3,−5) is on the graph of , what point must be on the graph of ?  Option #1: (3,5)  Option #2: (−3,−5)  Option #3: (−3,5)  The point in Option #\_\_\_\_ must be on the graph of . | 2 |
| P 5 | Use the image to answer the question.  Four concave curves are drawn on a coordinate plane with the x-axis from negative 6 to 6 and the y-axis from negative 4 to 4, both in increments of 2.  Given the graph of , which of the options is the graph of ?  The graph of is option \_\_\_\_. | 1 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Use the image to answer the question.  A parabola is drawn on a coordinate plane. Both axes range from negative 5 to 5 in one-unit increments.  The original graph of the function  is shown. Select the table that represents . |  |
| Q 2 | If the function is reflected horizontally, how does the reflection affect the point (−2,1)? | In the horizontal reflection, the point (−2,1) becomes (2,1). |
| Q 3 | Choose the correct statement to describe a true relationship between any point on  and . | If (x,y) is on the graph of ., then (−x,y) is on the graph of |
| Q 4 | Use the image to answer the question.  Four lines with bidirectional arrows are plotted and labeled on a coordinate plane. The equation f left parenthesis x right parenthesis equals x plus 1 appears at the top of the graph in quadrant 1.  Given the graph of , which option is the graph of ? | The graph of is option 3. |
| Q 5 | Use the image to answer the question.  Five labeled sinusoidal waves are plotted on a coordinate plane.  Describe the transformation of the graph of to the graph labeled Option 3. | Option 3 is the graph of . |

**Lesson 5 – Reflections About the Origin**

**Key Words:**

* **horizontal reflection** – a reflection of the graph of a function over the y-axis
* **reflection** – a transformation that reflects the graph of a function over a horizontal or vertical line
* **vertical reflection** – a reflection of the graph of a function over the x-axis

**Formulas:**

* Reflection about the origin equation:
* Reflection about the origin coordinates:

**Objective 1:** In this section, you will use tables and graphs to show how results in the function being reflected over both the *y*- and *x*-axes, otherwise known as a reflection about the origin.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* [Recall](#Bookmark2) vertical and horizontal reflections.
* When a *reflection* occurs over **both** the *x-* and *y-*axes, it is called a **reflection about the origin**, which has the coordinates *(0, 0)*.
  + A reflection about the origin combines the rules for a *vertical* and *horizontal reflection*. Both the *x-* and *y-*values are changed to the opposite.
  + Equation:
  + Coordinate Points:

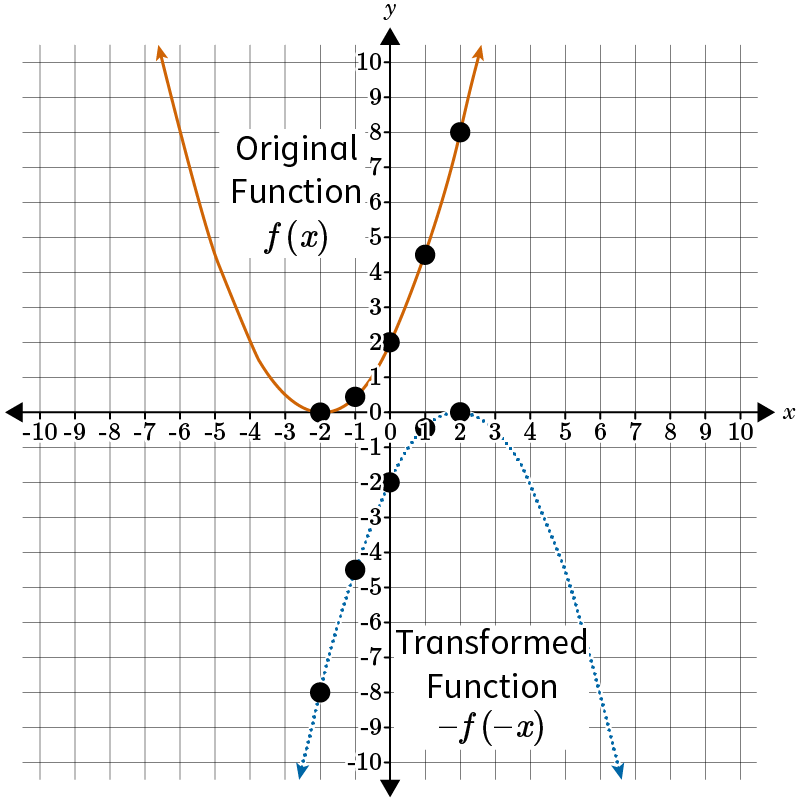
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| **Example:** Create a table for the function given the following table for . | |
| **Step 1:** Identify the transformation. | indicates a reflection about the origin. This means that each input and output value will be opposite of the original input value . |
| **Step 2:** Create a table for the function . |  |

**Objective 2:** In this section, you will describe the transformation from to function graphs of the form .

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* When looking at the graph of a *parent function* (original function) that has been *reflected about the origin* you should notice:
  + The new function is created by reflecting the original function over both the *x-*axis and the *y-*axis.
    - You can visualize this as two steps: flipping the image upside-down, and then reflecting it over the *y-*axis.
  + All of the *x-* and *y-*values become their opposites because all of the inputs and outputs are multiplied by -1.



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| **Example:** Identify which transformation on the graph (A, B, or C) represents as a reflection of . | |
| **Step 1:** Look at the location. | **Graph C** is the reflection about the origin of because the transformed function is flipped over the *x-* and *y*-axes. |
| **Step 2:** Look at the points. | The *x-* and *y-*coordinates are negated (switch signs) when the function flips over the *x-* and *y-*axes.   |  |  | | --- | --- | |  | *Graph C* | | (-1, 1) | (1, -1) | | (-3, 1) | (3, -1) | |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Create a table for the reflection about the origin of the function .  Hint: First create a table of values for  evaluated at x=0, x=−1, and x=−2. | -5; -2; -1 |
| P 2 | Reflect the function  about the origin. What is the position of the point (−4,9) in the reflected function?  The point (−4,9) reflects to (\_\_\_, \_\_\_) in | 4; -9 |
| P 3 | Complete the table to show that h(x) is the reflection about the origin of the function f(x). | -1; -1; -2; -8 |
| P 4 | If the function  is reflected about the origin, which of the following is true regarding the number of x-intercepts of the reflected function?  Hint: Sketch a graph of the function , then sketch a graph of the function reflected about the origin.  Option #1: 0 x-intercepts  Option #2: 1 x-intercept  Option #3: 2 x-intercepts | 3 |
| P 5 | Use the image to answer the question.  Four concave curves are drawn on a coordinate plane with the x-axis from negative 6 to 6 and the y-axis from negative 4 to 4, both in increments of 2.  Given the graph of , which option shows the reflection of the graph about the origin?  The reflection of the graph of about the origin is option \_\_\_\_. | 2 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Use the image to answer the question.  A parabola is drawn on a coordinate plane. Both axes range from negative 5 to 5 in one-unit increments.  The function  is shown in the graph. Which table shows that  results in a reflection about the origin of the function graphed?  Hint: Make a table of the x- and y-coordinates based on the graph. Use these points to determine the coordinates of the function reflected about the origin. |  |
| Q 2 | Which of the following is the vertex of  if the original function has a vertex of (−9,−8)? | (9, 8) |
| Q 3 | Choose the statement that **best** describes a true relationship between any point on  and | If (x,y) is on the graph of , then (−x,−y) is on the graph of |
| Q 4 | If the point (7,43) is on the graph of , what point must be on the graph of ? | (-7, -43) |
| Q 5 | Use the image to answer the question.  Five labeled sinusoidal waves are plotted on a coordinate plane.  Describe the transformation of the graph of  to the graph labeled Option 2. | Option 2 is the graph of |

**Lesson 6 – Even & Odd Functions**

**Key Words:**

* **even function** – a function that returns the same expression for both x and −x; this means for all values of *x*
* **odd function** – a function such that

**Formulas:**

* Even functions:
* Odd functions:

**Objective 1:** In this section, you will identify functions that look the same after being reflected about the *y*-axis as even functions.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* *Even functions* return the same expression for both *x* and *–x*.
  + is an even function when
  + The output, or *y-*value, will be the same if the *x-*value is positive or negative.
  + When represented in a table, the and values are identical.
  + When reflected on a graph, the reflection is symmetric about the *y-*axis.
* The quadratic function, is an example of an even function:
  + Notice that, when *x* is –2, the output is the same as when *x* is +2.

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| *x* |  |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |

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| **Example:** Explain why the following graph is **not** an even function. | |
| **Step 1:** Examine the graph. | The graph shows that the function is **not** symmetrical about the *y-*axis. The axis of symmetry, or middle of the function, is not along the *y-*axis. Therefore, the function is not even. |
| **Step 2:** Show the points in a table. | The table shows that the *x* and *–x* values do not generate the same values when plugged into the function.   |  |  | | --- | --- | | ***x*** | ***f(x)*** | | -2 | 2 | | -1 | 0 | | 0 | 0 | | 1 | 2 | | 2 | 6 | |
| **Step 3:** Look at the equation. | If the function is even, then will produce the same function. |

**Objective 2:** In this section, you will be able to identify functions that look the same after being reflected about the origin as an odd function.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* *Odd functions* are symmetric when reflected about the origin.
  + is an *odd function* when
* *Odd functions* can be determined algebraically using the steps in this example.

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| **Example:** Algebraically determine if the function is odd by calculating whether . | |
| **Step 1:** Determine by substituting *–x* in place of *x* in the function. |  |
| **Step 2:** Determine by multiplying the entire function by –1. |  |
| **Step 3:** Determine if it is an odd function | Therefore, is satisfied and it is an odd function. |

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| **Example:** Explain why the graph is **not** an odd function. | |
| **Step 1**: Look at the graph. | * The function is **not an odd function** because the graph is not symmetrical about the origin. * The points on the opposite sides of the origin are not the same distance from the *x-*axis. |

**Practice Questions and Answers**

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| --- | --- | --- |
|  | Question | Answer |
| P 1 | Use the image to answer the question.  A parabola is graphed on a coordinate plane. The x-axis ranges from negative 5 to 5 in increments of 1. The y-axis ranges from negative 1 to 16 in increments of 1.  Use the graph to determine if the function  is an even function.  Option #1: Yes, the function is an even function.  Option #2: No, the function is not an even function. | 1 |
| P 2 | Use the image to answer the question.  A parabola is graphed on a coordinate plane. The x-axis ranges from negative 4 to 6 in increments of 1. The y-ais ranges from negative 8 to 4 in increments of 1.  Using the graph, determine if the function is an even function. Select the correct option.  Option #1: Yes, the function is an even function.  Option #2: No, the function is not an even function. | 2 |
| P 3 | Use the table to answer the question.    If f(x) is an even function, what is the value of the missing entry in the table?  If f(x) is even, the missing entry is \_\_\_\_. | -2 |
| P 4 | Sketch the graphs of  and . Based on the graphs, is  an even function, an odd function, or neither even nor odd?  Option 1: even function  Option 2: odd function  Option 3: neither even nor odd function  The option that describes the function  is Option \_\_\_\_. | 2 |
| P 5 | The graph of the function  looks the same when it is reflected about the origin. Is an even function, an odd function, or neither even nor odd function?  Option 1: even function  Option 2: odd function  Option 3: neither even nor odd function  The option that describes the function  is Option \_\_\_\_. | 2 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Identify the graph of an even function from the following choices. | Five points are plotted on a coordinate plane, and a line is graphed passing through them. The x-axis ranges from negative 4 to 4 in increments of 1. The y-axis ranges from negative 10 to 10 in increments of 1. |
| Q 2 | Which table provides points for an even function? |  |
| Q 3 | If  is an even function and the point  is on the graph of , what other point must be on the graph of ? |  |
| Q 4 | Identify the odd function. |  |
| Q 5 | Which table shows an odd function? |  |

**Lesson 7 – Vertical Shifts**

**Key Words:**

* **transformation** – a manipulation of a function so that its graph is translated, reflected, rotated, or dilated
* **translation** – a transformation that shifts the graph of a function vertically or horizontally
* **vertical shift** – a translation that shifts the graph of a function vertically

**Formulas:**

* Vertical Shift equation:
* Vertical Shift coordinates:

**Objective 1:** In this section, you will use tables and graphs to show how results in the function being translated up or down *k* units.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* [Recall](#Bookmark2) that a *vertical shift* is a *translation* of a function upward or downward.
* Vertical shifts change the equation of a function:
  + shifts the graph **up** by *k* units.
  + shifts the graph **down** by *k* units.

|  |  |
| --- | --- |
| **Example:** Use a table and a graph to show that the function is translated 3 units down from the parent function . | |
| **Step 1:** Create an input-output table for the original parent function . Show the coordinate pairs. | |  |  |  | | --- | --- | --- | | ***x*** |  | ***coordinate*** | | -2 |  | (-2, -2) | | -1 |  | (−1, −1) | | 0 |  | (0, 0) | | 1 |  | (1, 1) | | 2 |  | (2, 2) | |
| **Step 2:** Plot the coordinates, connect the points, and label the graph. |  |
| **Step 3:** Create an input-output table for the transformed function . Show the coordinate pairs. | |  |  |  | | --- | --- | --- | | ***x*** |  | ***coordinate*** | | -2 |  | (−2, −5) | | -1 |  | (−1, −4) | | 0 |  | (0, −3) | | 1 |  | (1, −2) | | 2 |  | (2, −1) | |
| **Step 4:** Plot the coordinates, connect the points, and label the graph. |  |
| **Step 5:** State the answer. | The graph is a vertical translation, 3 units down from the graph . |

**Objective 2:** In this section, you will identify *k* and the effect it has on a given function from graphs and equations that show vertical shifts.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

* [Recall](#Bookmark3) how a vertical shift changes the parent function:
* To identify the *vertical shift k* and the effect of the shift on a *parent function*, use the following steps:
  + 1. Compare the equation of the given function and its *parent function*. Determine differences.
  + 2. Compare the graph of the given function and its *parent function*. Determine differences.
  + 3. Use the results from steps 1 and 2 to identify *k* and the effect of the *vertical shift* on the *parent function*.

|  |  |
| --- | --- |
| **Example:**  Use the graph and the equation to identify the vertical shift *k* and the effect it has on the parent function. | |
| **Step 1:** Compare the equation of the given function and its *parent function*. Determine differences | The parent function of is .  Notice that the difference in the equation is +2; therefore, . |
| **Step 2:** Compare the graph of the given function and its *parent function*. Determine differences. | Notice that the graph of is up 2 units from . |
| **Step 3:** Use the results from steps 1 and 2 to identify *k* and the effect of the *vertical shift* on the *parent function*. | * The function has a vertical shift of . * The effect is that the graph moves up 2 units from the parent function . * There is no change in the size or shape of the graph. |

**Objective 3:** In this section, you will describe the transformations from  to function graphs of the form .

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* [Recall](#Bookmark3) how a *vertical shift* changes the parent function:
* When the *parent function* is shifted up or down, **only the *y-*coordinate** of the transformed function is affected.
  + : shift up and the *y-*coordinate will increase by *k*.
  + : shift down and the *y-*coordinate will decrease by *k*.

|  |  |
| --- | --- |
| **Example:** Examine the graph and coordinate points of the parent function . If a **vertical shift of** is applied to the parent function, find the new coordinates and create the graph of the transformed function. | |
| **Step 1:** List the coordinates of the parent function. | |  |  |  | | --- | --- | --- | |  |  | Coordinate | | -2 | -2 | (-2, -2) | | -1 | -1 | (-1, -1) | | 0 | 0 | (0, 0) | | 1 | 1 | (1, 1) | | 2 | 2 | (2, 2) | |
| **Step 2:** Apply the coordinate rule for vertical shifts to the parent function coordinates. | where   |  |  |  | | --- | --- | --- | | Parent Function Coordinate |  | Transformed Coordinate | | (-2, -2) |  | (-2, 2) | | (-1, -1) |  | (-1, 3) | | (0, 0) |  | (0, 4) | | (1, 1) |  | (1, 5) | | (2, 2) |  | (2, 6) | |
| **Step 3:** Plot the coordinate points of the transformed function to create the graph. |  |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Use the image to answer the question.  Two lines are graphed on a coordinate plane. The x-axis ranges from negative 3 to 3 in increments of 1. The y-axis ranges from negative 2 to 2 in increments of 1.  The graphs of the functions  (solid line) and  (dotted line) are shown. How does adding 2 to  impact the graph?  Option 1: It translates the graph of  up 2 units.  Option 2: It translates the graph of  down 2 units.  Option 3: It reflects the graph of  over the x-axis and translates it up 2 units. | 1 |
| P 2 | Two tables were generated to compare the functions  and . Find the missing value in the table for the function | 0 |
| P 3 | Use the image to answer the question.  Two S-shaped curves are graphed on a coordinate plane. The x-axis ranges from negative 3 to 3 in increments of 1. The y-axis ranges from negative 5 to 5 in increments of 1.  The graph shows a parent function  (solid line) and its translated form  (dotted line) which has undergone a vertical shift. Use the graph to find the value of *k*.  *k*= \_\_\_\_ | -2 |
| P 4 | Use the image to answer the question.  Two V-shaped curves are graphed on a coordinate plane. The x-axis ranges from negative 3 to 3 in increments of 1. The y-axis ranges from negative 5 to 5 in increments of 1.  The graph shows a parent function as a solid line and the translated function as a dotted line. Describe the effect the transformation has on the equation of the parent function  by determining the value of *k*in the translated function.  The value of *k* in the translated function is  + \_\_\_\_\_. | 3 |
| P 5 | Use the image to answer the question.  Two parabolas are graphed on a coordinate plane. The x-axis ranges from negative 3 to 3 in increments of 1. The y-axis ranges from negative 5 to 5 in increments of 1.  The function graphed with a blue dotted line is translated to the function graphed as a solid orange line. If the same translation were applied to the solid orange function, what would be the coordinates of the new *y*-intercept (not pictured)?  The new *y*-intercept would be (\_\_\_\_, \_\_\_\_). | 0; 3 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Which statement best shows how the vertical shift affects the parent function  when shifted to ? | will have each point 5 steps higher than . |
| Q 2 | Which graph accurately demonstrates the relationship between the functions  and ? | Two curves are graphed on a coordinate plane. The x-axis ranges from negative 3 to 3 in increments of 1. The y-axis ranges from negative 6 to 6 in increments of 2. |
| Q 3 | Determine the x-intercept(s) of the function that translates the original function  down 4 units. | x=−2 and x=2 |
| Q 4 | Describe the transformation that occurs on a graph when changing an equation from  to . | The original function shifts vertically up four units. |
| Q 5 | Use the images to answer the question.    The image shows a parent function  and a translated form . Which statement **best** describes this translation? | The parent function shifted vertically down 3 units and did not change in any other way. |

**Lesson 8 – Horizontal Shifts**

**Key Words:**

* **horizontal shift** – a translation that shifts the graph of a function horizontally
* **transformation** – a manipulation of a function so that its graph is translated, reflected, rotated, or dilated
* **translation** – a transformation that shifts the graph of a function vertically or horizontally
* **vertical shift** – a translation that shifts the graph of a function vertically

**Formulas:**

* Horizontal Shift equation:

**Objective 1:** In this lesson, you will use tables and graphs to show how results in the function being translated left or right *k* units.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* [Recall](#Bookmark2) that a *horizontal shift* is a *translation* of a function left or right by *k* units.
* Horizontal shifts change the equation of a function:
  + : horizontal shift **right**
  + : horizontal shift **left**

|  |  |
| --- | --- |
| **Example:** Use a table and a graph to show that the function has been translated from its parent function . | |
| **Step 1:** Create an input-output table for the original parent function . Show the coordinate pairs. | |  |  |  | | --- | --- | --- | | ***x*** |  | ***coordinate*** | | -2 |  | (−2, 4) | | -1 |  | (−1, 1) | | 0 |  | (0, 0) | | 1 |  | (1, 1) | | 2 |  | (2, 4) | |
| **Step 2:** Plot the coordinates, connect the points, and label the graph. |  |
| **Step 3:** Create an input-output table for the transformed function . Show the coordinate pairs. | |  |  |  | | --- | --- | --- | | ***x*** |  | ***coordinate*** | | -2 |  | (−2, 1) | | -1 |  | (−1, 0) | | 0 |  | (0, 1) | | 1 |  | (1, 4) | | 2 |  | (2, 9) | |
| **Step 4:** Plot the coordinates, connect the points, and label the graph. |  |
| **Step 5:** State the answer. | The graph of is a horizontal translation that is shifted 1 unit to the left of the graph of . |

**Objective 2:** In this lesson, you will identify *k* and the effect it has on a given function from graphs and equations that show horizontal shifts.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

* [Recall](#Bookmark4) the rules for *horizontal shifts:* 
  + If the shift is to the **left** *k* units.
  + If the shift is to the **right** *k* units.
* To identify the *horizontal shift*, *k*, and the effect of the shift on the *parent function*, use the following steps:
  + 1. Compare the equation of the given function and its parent function. Determine differences.
  + 2. Compare the graph of the given function and its parent function. Determine differences.
  + 3. Use the results from Step 1 and Step 2 to identify *k* and the effect of the *horizontal shift* on the *parent function*.

|  |  |
| --- | --- |
| **Example:** Use the graph and the equation to identify the horizontal shift, k, and the effect it has on the parent function . | |
| **Step 1:** Compare the equation of the given function and its parent function. Determine differences. | Parent function:  Translated function:  The translated function has 3 subtracted from the variable *x* inside the function.  can be written as . |
| **Step 2:** Compare the graph of the given function and its parent function. Determine differences. | The two graphs have the same size and shape but one is shifted to the right of the other. |
| **Step 3:** Use the results from Step 1 and Step 2 to identify *k* and the effect of the *horizontal shift* on the *parent function*. | Compare the function to . Written in this form, you can identify the value of *k* as 3.  Comparing the graphs, you can see that the graph of the translated function has moved to the right 3 units from the parent function.  There is no change in the size or the shape of the graph. |

**Objective 3:** In this lesson, you will describe the transformations from  to function graphs of the form .

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* Recall that a *horizontal shift* moves an object on a graph left or right without changing its size or shape.
* It is possible to translate functions that are not parent functions.
  + Once the original function is graphed, map each point to its new location using the translation rule.

|  |  |
| --- | --- |
| **Example 1:** The following graph represents the function . If this function is translated to the function , how has the function moved? | |
| **Step 1:** Identify the translation that has occurred by comparing the original function and the translated function. | From to , the difference between 5 and 3 is 2.  This means that 2 has been added to the *x* variable, . |
| **Step 2:** Recall the rule for horizontal translations. | When *k* is added to the *x* variable, a horizontal shift to the left occurs by *k* units.  In this case, , has been added to the *x* variable. |
| **Step 3:** State the answer. | Each point on the graph is moved two units to the left. |

|  |  |
| --- | --- |
| **Example 2:** The function is translated 5 units to the right. What is the equation of the translated function, and where will the point (4,3) be mapped to? | |
| **Step 1:** Find the equation of the translated function. | If is moved 5 units to the right: |
| **Step 3:** Map the new coordinate. | The original coordinate is (4,3). The coordinate is moved 5 units to the right. Add 5 to the *x*-coordinate or count 5 to the right on the graph. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Complete the tables to show the parent function  and its translation . | 1. -7 2. -6 3. -5 4. -4 5. -3 |
| P 2 | Use the tables to answer the question.    If  is the original function and  the horizontally translated function, by how many k units is the original function translated? | 4 |
| P 3 | Use the images to answer the question.  **Original function:**  A coordinate plane's x-axis ranges from negative 9 to 9 and its y-axis ranges from negative 11 to 11, both by 1-unit increments. A concave up parabola labeled f of x equals x squared is plotted passing through 7 marked points.  **Translated function:**  A coordinate plane's x-axis ranges from negative 9 to 9 and its y-axis ranges from negative 11 to 11, both by 1-unit increments. A parabola of the function f left parenthesis x parenthesis is plotted.  Notice the horizontal shift in the images. What is the value of k for the horizontal shift that occurred between the graph of the original function and the graph of the translated function ? | -1 |
| P 4 | Use the images to answer the question.  **Original function:**  A V shaped function with arrows at both ends passes through 7 plotted points. The x-axis ranges from negative 9 to 9 in unit increments and the y-axis ranges from negative 11 to 11 in unit increments.  **Translated function:**  A V shaped function with arrows at both ends passes through 7 plotted points. The x-axis ranges from negative 9 to 9 in unit increments and the y-axis ranges from negative 11 to 11 in unit increments.  How far has the original function shifted horizontally? | 2 |
| P 5 | Write an equation that describes the function  shifted to the left 3 units.  \_\_\_ | 4; 0 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Each of the function graphs represents an original function, marked by a dotted line graph, and a translated graph, marked by a solid line graph. Which graph shows that  is the result of the function  being translated by 3 units? | Two lines drawn on coordinate plane. Both axes range from negative 5 to 5 in unit increment. |
| Q 2 | Use the image to answer the question.  Two parabolas drawn on coordinate plane. X-axis range from negative 9 to 4 and y-axis range from negative 1 to 9 in unit increment.  Which of the following correctly determines how many *k* units the parent function  (shown as a solid line) was translated? | The parent function was translated 5 units to the left. |
| Q 3 | Use the image to answer the question.  A coordinate plane labeled 'Horizontal Shift in the Demand Curve' has axes ranging from 0 to 50, both by 5-unit increments. The x-axis is labeled 'Number of Units' and the y-axis is labeled 'Price' with dollar units. Two line segments are plotted.  Which of the following identifies the value of k that would correctly characterize the horizontal shift shown in the graph if the line to the left represents  and the line to the right represents ? | 10 |
| Q 4 | Which of the following **best** describes the transformation that occurs when changing an equation from  to ? | The graphed function would shift 3 units to the right but otherwise have the same shape. |
| Q 5 | Use the image to answer the question.  Two curves drawn on coordinate plane. Both axes range from negative 5 to 5 in unit increment.  The original function  in the graph (solid line) goes through the point (1,1) and is translated using a horizontal translation. Where is this point located after the translation, and what is the translated function? | The point is located at (3,1), and the translated function is . |

**Lesson 9 – Vertical Stretches & Compressions**

**Key Words:**

* **dilation** – a transformation that stretches or compresses the graph of a function horizontally or vertically
* **transformation** – a manipulation of a function so that its graph is translated, reflected, rotated, or dilated
* **vertical compression** – a dilation that compresses the graph of a function vertically
* **vertical stretch** – a dilation that stretches the graph of a function vertically

**Formulas:**

* Vertical Dilation equation:
  + : compression
  + : stretch
* Vertical Dilation coordinate:

**Objective 1:** In this section, you will use tables and graphs to show that results in the function being vertically stretched or compressed by a factor of *k*.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* [Recall](#Bookmark2) that vertical stretches and compressions change the equation from: to
  + *: vertical compression* (or shrinking) by a factor of *k.*
  + : *vertical stretch* by a factor of *k*.
* When graphed, the *y-*coordinate of the translated function is multiplied by a factor of *k*.

|  |  |
| --- | --- |
| **Example:** Use a table and a graph to show the vertical stretch of compared to its parent function . | |
| **Step 1:** Create an input-output table for the parent function. Include the coordinates. | |  |  |  | | --- | --- | --- | | ***x*** | ***Evaluate*** | ***Coordinate*** | | -2 |  | (−2, 4) | | -1 |  | (−1, 1) | | 0 |  | (0, 0) | | 1 |  | (1, 1) | | 2 |  | (2, 4) | |
| **Step 2:** Graph the parent function. |  |
| **Step 3:** Create an input-output table for the transformed function. Include the coordinates. | |  |  |  | | --- | --- | --- | | ***x*** | ***Evaluate*** | ***Coordinate*** | | -2 |  | (−2, 8) | | -1 |  | (−1, 2) | | 0 |  | (0, 0) | | 1 |  | (1, 2) | | 2 |  | (2, 8) | |
| **Step 4:** Graph the translated function. |  |
| **Step 5:** State answer. | The graph is a vertical stretch by 2 from the graph of .   * Note that the graph of the dilated function is stretched such that the points are farther away from the *x*-axis. * Each *y-*coordinate in the dilated table has been multiplied by 2 when compared to the *y*-coordinates in the table for the parent function. |

**Objective 2:** In this section, you will identify *k* and the effect it has on a given function from graphs and equations that show vertical stretches or compressions.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

* [Recall](#Bookmark5) the rules for *vertical stretches and compressions.*
* To identify the factor *k* and its effect on the *parent function*, use the following steps:
  + 1. Compare the equation of the given function and its parent function. Determine differences.
  + 2. Compare the graph of the given function and its parent function. Determine differences.
  + 3. Use the results from Step 1 and Step 2 to identify *k* and its effect on the parent function.

|  |  |
| --- | --- |
| **Example:** Use the graph and the equation to its parent function . | |
| **Step 1:** Compare the equation of the given function and its parent function. Determine differences. | Parent Function:  Translated Function:  Notice that the difference in the equation is ; therefore, . |
| **Step 2:** Compare the graph of the given function and its parent function. Determine differences. | The graph has moved, or compressed, closer to the *x-*axis . |
| **Step 3:** Use the results from Step 1 and Step 2 to identify k and its effect on the parent function. | The function is a vertical shrink or compression where .   * The effect is that the graph is compressed by a factor of from the parent function . * The graph is compressed closer to the *x-*axis than the graph of its parent function. |

* What happens when the original function is **not a *parent function***?

|  |  |
| --- | --- |
| **Example:** Dilate the linear function vertically by a factor of . | |
| **Step 1:** Use the rule for vertical dilations. | The factor is . |
| **Step 2:** Apply the rule to the original function. |  |
| **Step 3:** Compare the original function and the translated function on a graph. | The slope of the translated function, , is steeper by a factor of 2. |

**Objective 3:** In this section, you will describe the transformations from  to function graphs of the form .

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* [Recall](#Bookmark5) the rules for *vertical dilations*.
* When an original function undergoes a vertical dilation, the coordinate points are affected.
  + The *y-*coordinates are multiplied by the factor *k:* 
    - Vertical Stretch: each point is further away from the *x-*axis
    - Vertical Compression: each point is closer to the *x-*axis
* All functions, not just parent functions, can be dilated.

|  |  |
| --- | --- |
| **Example:** The graph of the function is vertically dilated by scale factor . Compare the coordinate points of the original function and the transformed function. | |
| **Step 1:** Create an input-output table for the original function. Include coordinate points. | |  |  |  | | --- | --- | --- | | ***x*** | ***Evaluate*** | ***Coordinate*** | | -1 |  | (-1,9) | | 0 |  | (0,4) | | 1 |  | (1,1) | | 2 |  | (2,0) | | 3 |  | (3,1) | |
| **Step 2:** Write the transformed function. using the rules for vertical dilations. |  |
| **Step 3:** Create an input-output table for the transformed function. Include coordinate points. | |  |  |  | | --- | --- | --- | | ***x*** | ***Evaluate*** | ***Coordinate*** | | -1 |  | (-1,36) | | 0 |  | (0,16) | | 1 |  | (1,4) | | 2 |  | (2,0) | | 3 |  | (3,4) | |
| **Step 4:** Compare the original points to the transformed points. | * The *x*-coordinate stays the same. * The *y*-coordinate is multiplied by a factor of  |  |  | | --- | --- | | **Original** | **Transformed** | | (-1,9) | (-1,36) | | (0,4) | (0,16) | | (1,1) | (1,4) | | (2,0) | (2,0) | | (3,1) | (3,4) | |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Complete the table to vertically stretch the parent function  by a factor of 3. | 3; 3; 24 |
| P 2 | Complete the tables to show that  results in the parent function being vertically compressed when | ; 8 |
| P 3 | Use the image to answer the question.  Two open, downward facing parabolas of different sizes that share a vertex are plotted on a coordinate plane. 1 is a solid line and 1 is a dotted line.  Identify the k-value for the vertical stretch from the original function (the solid parabola) to the dilated function (the dotted parabola). | 3 |
| P 4 | Identify the factor k for the vertical compression from the original function  to the dilated function  Enter your response as a fraction. |  |
| P 5 | Consider the following two functions.  **Original:**  **Final:**  Which option **best** describes the change from the original to the final function?  Option #1: a vertical stretch  Option #2: a vertical compression  Option #3: no change  The change is best described by Option # \_\_\_\_\_. | 2 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Which of the following tables shows that  results in the parent function  being vertically stretched by a k factor of 3? |  |
| Q 2 | Use the image to answer the question.  A coordinate plane shows two v-shaped curves.  The function graphed with a solid line is the parent function,  Which of the following describes the transformed function, , which is shown with a dotted line? | The parent function has been vertically stretched by a factor of 4. |
| Q 3 | Which of the following correctly identifies the factor k for the vertical compression from the function  to the function ? |  |
| Q 4 | Use a graphing calculator to graph the original function  and the vertically dilated function with a factor . Determine the x-intercepts of both functions. | The original function has an *x*-intercept of (8,0), and the dilated function has an *x*-intercept of (8,0). |
| Q 5 | Use the images to answer the question.    Both graphs represent transformations from their respective parent functions. Describe the transformation. | Both graphs represent vertical compressions, although they use different factors of *k*. |

**Lesson 10 – Horizontal Stretches & Compressions**

**Key Words:**

* **dilation** – a transformation that stretches or compresses the graph of a function horizontally or vertically
* **function** – an expression, rule, or law that defines a relationship between the independent variable and the dependent variable
* **horizontal compression** – a dilation that laterally compresses the graph of the function; occurs when the x-value of a function is multiplied by a constant, *k*, whose value is greater than 1
* **horizontal stretch** –a dilation that laterally stretches the graph of a function; occurs when the -value of a function is multiplied by a constant, *k*, whose value is between 0 and 1
* **transformation** – a manipulation of a function so that its graph is translated, reflected, rotated, or dilated

**Formulas:**

* Horizontal Dilation equation:
  + : compression
  + : stretch
* Calculate *k* value:

**Objective 1:** In this section, you will use tables and graphs to show how will horizontally stretch or compress (dilate) by a factor of *k*.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* [Recall](#Bookmark2) that horizontal stretches and compressions change the equation from: to
  + *: horizontal compression* by a factor of *k.*
  + : horizontal *stretch* by a factor of *k*.
* To determine the mapping of new points in a horizontal dilation, multiply the inputs of the function by a factor of *k* so that the dilated function becomes .

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Example:** The function *g(x)* is created by horizontally stretching the original function , with a *k* factor of 2.  Given the table of the original function *f(x)*, complete a table for *g(x)* to show how the functions changed.   |  |  | | --- | --- | | ***x*** | **Original**  ***f(x)*** | | -1 | -18 | | 0 | -2 | |  | 0 | | 1 | -2 | | 2 | -18 | | |
| **Step 1:** Determine the new function. | Original Function:  Horizontal **stretch** by a factor of 2.   * *k* must be between for a horizontal stretch to occur * To stretch by a factor of 2, **multiply the variables** in the original function by   Transformed function: |
| **Step 2:** Create a table for *g(x).* | |  |  |  | | --- | --- | --- | | ***x*** | ***Evaluate*** | ***g(x)*** | | -1 |  | -8 | | 0 |  | -2 | |  |  | -0.5 | | 1 |  | 0 | | 2 |  | -2 | |

**Objective 2:** In this section, you will identify *k* and the effect it has on a given function from graphs and equations that show horizontal stretches or compressions.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

* [Recall](#Bookmark2) that *horizontal stretches* and *compressions* change the equation from:
  + If , then a *horizontal stretch* occurs.
  + If , then a *horizontal compression* occurs.
* A *horizontal dilation* can be compared to changing the inputs of the function to obtain the same output.

|  |  |
| --- | --- |
| Identify the value of *k* and determine if the graph would be a horizontal stretch or compression. | |
| **Example 1:** The function is transformed to .   * ; horizontal stretch * For the same output value (*y*), the input value (*x*) is doubled. | **Example 2:** The function is transformed to .   * ; horizontal compression * For the same output value (*y*), the input value (*x*) is as large. |

* If the **input value (*x*) changes for the same output value (*y*)**, the transformation is a *horizontal stretch or compression*.

|  |  |
| --- | --- |
| **Example:** Suppose is transformed to . The *x* is multiplied by 2. This means the input values of the function have been transformed.  Create two tables to compare how the input values change for the **same output value.** | |
| **Step 1:** Determine the value of *k*. | Recall:  The value of *k* is 2, and is a horizontal compression compared to *f(x).*   * Recall that when *k* is greater than 1, a horizontal compression occurs * For the same output value (*y*), the input value (*x*) is half. |
| **Step 2:** Create a table for *f(x)*. | |  |  | | --- | --- | | ***x*** | ***f(x)*** | | -2 | -1 | | -1 | -2 | | 0 | -3 | | 1 | -2 | | 2 | -1 | |
| **Step 3:** Create a table for *using the same output values.* | For the same output value (*y*), the input value (*x*) is **half**.   |  |  | | --- | --- | | ***x*** | ***f(2x)*** | | -1 | -1 | | -0.5 | -2 | | 0 | -3 | | 0.5 | -2 | | 1 | -1 | |

**Objective 3:** In this section, you will describe the transformations from  to function graphs of the form .

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* [Recall](#Bookmark2) that *horizontal stretches* and *compressions* change the equation from:
  + Ifis , then a *horizontal stretch* occurs.
  + If , then a *horizontal compression* occurs.
* The exact value of *k* may be determined by comparing points on the graph.
  + 1. Select a point on the original graph with an integer *x-*value and in integer *y-*value.
  + 2. Select the point on the transformed graph that has the **same *y-*value** as that of the original graph.
    - Note: If the point on the transformed graph doesn’t have an *integer* *x-*value, start the process again from Step 1.
  + 3. Calculate the *k* value:

|  |  |
| --- | --- |
| **Example:** Describe the transformation from to the unction graph of the form .  : solid orange graph  : dotted blue graph | |
| **Step 1:** Select a point on the original graph with an integer *x-*value and in integer *y-*value. | (1, 1) |
| **Step 2:** Select the point on the transformed graph that has the **same *y-*value** as that of the original graph. | (4, 1) |
| **Step 3:** Calculate *k*. | The value of *k* is , which is between 0 and 1 as defined for a horizontal stretch. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | The parent function  is horizontally stretched by a factor of 3. Points on the parent function include (0,0), (1,1), (2,4), and (3,9). Complete the table for | 6; 9 |
| P 2 | Use the tables to answer the question.   The function  is created by horizontally stretching the parent function,  with a k factor of . Complete the table to show how the function has changed. | -3; 4.5; 6 |
| P 3 | From the equations  and , identify whether the graph shows a compression or a stretch.  Type 1 for compression.  Type 2 for stretch. | 1 |
| P 4 | Use the image to answer the question.  A solid curve and a dotted curve are plotted on a coordinate plane. Both curves begin in quadrant 2 sloping downward from left to right, pass briefly through quadrant 3, and then descend through quadrant 4.  Equations  and  are graphed. Identify whether the graph shows a compression or a stretch.  Type 1 for compression.  Type 2 for stretch. | 2 |
| P 5 | The following ordered pairs came from a function transformation. The original ordered pair is (8,2), and the transformation ordered pair is (2,2). Identify the value of k. | 4 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Which graph shows the parent function  horizontally compressed to form the graph of ? | An upward parabola passes through 3 plotted points. The x-axis ranges from negative 5 to 5 in 1-unit increments and the y-axis ranges from 0 to 10 in 1-unit increments. |
| Q 2 | Use the image to answer the question.  A upward facing V-shaped graph is plotted on a coordinate plane. The x-axis ranges from negative 3 to 3 in 1-unit increments and the y-axis ranges from 0 to 3 in 1-unit increments.  Which of the following statements correctly describes how the parent function  was dilated to result in the graphed function? | The parent function was horizontally stretched. |
| Q 3 | The equation  is transformed to . Identify the value of k. Does the graph show a stretch or a compression? | 2; compression |
| Q 4 | Use the image to answer the question.  Two parabolas that open upward are plotted on a coordinate plane. Both parabolas begin in quadrant 2 sloping downward from left to right, pass through quadrant 3 until reaching a common vertex on the y-axis, and then rise through quadrants 4 and 1.  According to the graph, what is the value of k? Does the transformation show a stretch or a compression? | 0.5; stretch |
| Q 5 | Use the image to answer the question.  Two parabolas with a common vertex are drawn on a coordinate plane. The x-axis ranges from negative 3 to 3 in one-unit increments and the y-axis ranges from negative 4 to 2 in one-unit increments.  Which of the following **best** describes the transformation from the solid-line graph to the dotted-line graph and correctly states the factor k? | horizontal stretch with a factor |

**Lesson 11 – Multiple Transformations**

**Key Words:**

* **dilation** – a transformation that stretches or compresses the graph of a function horizontally or vertically
* **reflection** – a transformation that reflects the graph of a function over a horizontal or vertical line
* **translation** – a transformation where every point on the graph of a function moves by the same amount in the same direction

**Objective 1:** In this section, you will show using tables and graphs that combinations of transformations should be performed according to the order of operations.

*Mathematical Practice Standard: Use appropriate tools strategically.*

**Big Ideas**:

* Recall the rules and how functions can be transformed:

|  |  |  |
| --- | --- | --- |
| **Transformation** | **Rule** | **Result** |
| Horizontal Translation |  | Moves right if  Moves left if |
| Vertical Translation |  | Moves up if  Moves down if |
| Horizontal Dilation |  | Stretches if  Compresses if |
| Vertical Dilation |  | Compresses if  Stretches if |
| Horizontal Reflection |  | Reflects across *y-*axis |
| Vertical Reflection |  | Reflects across *x-*axis |

* Combinations of transformations should be performed according to the order of operations.
  + PEMDAS: Parentheses, Exponents, Multiplication or Division, Addition or Subtraction

|  |  |
| --- | --- |
| **Example:** Transform to . | |
| **Step 1:** Create a table of values to be plotted for the original function. | |  |  |  |  | | --- | --- | --- | --- | | ***x*** | ***Evaluate*** | ***f(x)*** | ***Coordinate*** | | -2 |  | -8 | (-2, -8) | | -1 |  | -1 | (-1, -1) | | 0 |  | 0 | (0, 0) | | 1 |  | 1 | (1, 1) | | 2 |  | 8 | (2, 8) | |
| **Step 2:** Create a table of values to be plotted for the transformed function. Calculate the values of the transformed function using the rules for PEMDAS. | |  |  |  |  |  | | --- | --- | --- | --- | --- | | ***x*** | **Step 1:**  Add two to the value of x | **Step 2:**  Cube the value from step 1 | **Step 3:**  Multiply the value from step 2 by 0.5 | **Coordinate** | | -2 | 0 | 0 | 0 | (-2, 0) | | -1 | 1 | 1 | 0.5 | (-1, 0.5) | | 0 | 2 | 8 | 4 | (0, 4) | | 1 | 3 | 27 | 13.5 | (1, 13.5) | | 2 | 4 | 64 | 32 | (2, 32) | |
| **Step 3:** Graph the original and transformed functions. |  |

**Objective 2:** In this section, you will identify in order the transformations undergone by a function from its equation.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

* [Recall](#Bookmark6) the rules of transformations and PEMDAS.
* When transformations have been combined, the order of transformations is related to the order of operations.
* Follow the order of operations to identify the order of transformations that occurred.

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| --- | --- |
| **Example:** Identify the order of transformations undergone from to . | |
| **Step 1:** Follow the order of operations. | * 1st: Parentheses – Multiply *x* by *4*. * 2nd: Exponents – Square *4x*. * 3rd: Add 6 to . |
| **Step 2:** Identify the order transformations that follow the order of operations. | * 1st: Compressed horizontally   + Horizontally because the value of *k* is in the form   + Compressed because the value of *k*, 4, is greater than 1. * 2nd: Translated vertically up   + Vertically because the value of *k* is in the form .   + Up because the value of *k*, 6 is greater than 0.   Note that the second order of operations, squaring 4*x*, is not a transformation because it is also part of the original function . |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | When using the order of operations to transform  to , what needs to be completed first?  Option #1: Square the expression (x−3).  Option #2: Multiply (x−3) by 2.  Option #3: Add 12 to (x−3). | 1 |
| P 2 | Complete the table with the order of operations needed to transform to . | 2; 1; 3 |
| P 3 | True or false: The order of the transformations undergone for  to  is translated horizontally to the left and then stretched vertically.  Type 1 for true.  Type 2 for false. | 1 |
| P 4 | True or false: The transformations undergone for  to  is a translation to the left 1 unit and a translation up 3 units.  Type 1 for true.  Type 2 for false. | 1 |
| P 5 | Cora is working on graphing . She starts by graphing  as the parent function. Next, she takes her graph of  and reflects it across the y-axis and calls the result .  True or false: In order to transform the graph of to make the graph , Cora should translate  three units to the right.  Type 1 for true.  Type 2 for false. | 2 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | To transform  to , which of the following shows the order in which operations should be performed? | Subtract 3 from x, square (x−3), and add 12. |
| Q 2 | What is the order of combinations to transform  to ? | Shift to the right 3 units and shift up 12 units. |
| Q 3 | Which of the following correctly transforms  to  for ? | 34 |
| Q 4 | Which of the following correctly identifies the transformations undergone from  to ? | compressed horizontally and shifted down |
| Q 5 | Which of the following correctly identifies the transformations undergone from  to ? | reflected across the y-axis and translated up vertically |

**Lesson 12 – Analyzing a Graph**

**Key Words:**

* **dilation** – a transformation that stretches or compresses the graph of a function horizontally or vertically
* **horizontal compression** – a dilation that laterally compresses the graph of the function; occurs when the *x*-value of a function is multiplied by a constant, *k*, whose value is greater than 1
* **horizontal reflection** – a reflection of the graph of a function over the *y*-axis
* **horizontal shift** – a translation that shifts the graph of a function horizontally
* **horizontal stretch** – a dilation that laterally stretches the graph of the function; occurs when the *x*-value is multiplied by a constant, *k*, whose value is between 0 and 1
* **parent function** – a function without transformations
* **reflection** – a transformation that reflects the graph of a function over a horizontal or vertical line
* **transformation** – a manipulation of a function so that its graph is translated, reflected, rotated, or dilated
* **translation** – a transformation where every point on the graph of a function moves by the same amount in the same direction
* **vertical compression** – a dilation that compresses the graph of a function vertically
* **vertical reflection** – a reflection of the graph of a function over the *x*-axis
* **vertical shift** – a translation that shifts the graph of a function vertically
* **vertical stretch** – a dilation that stretches the graph of a function vertically

**Formulas:**

* [Transformation Rules](#Bookmark6)

**Objective 1:** In this section, you will interpret the key features of a function graph to identify its type of transformation from the parent function.

*Mathematical Practice Standard: Use appropriate tools strategically.*

**Big Ideas**:

* [Recall](#Bookmark6) the rules for the different types of *transformations*.
* Recall that *transformations* do not change the general shape of a graph, but they do change its position and/or steepness of the curve.
* Sometimes, you are only given the transformed function without the *parent function*.
* Certain key features of the graph will help identify transformations from the parent function:
  + Shape of the graph – linear, U-shaped (quadratic), V shaped (absolute value), etc.
  + Slope – if the function is linear
  + x-intercepts
  + y-intercepts
  + how narrow or wide the graph is

|  |  |
| --- | --- |
| **Based on the key features of the graphed function, identify its type.** | |
| **Example 1:**   * The function is a quadratic function. * It has the same U-shape as the parent function, , but all coordinates have been shifted five units to the left. | **Example 2:**     * The function is a linear function. * It is a straight line like its parent function, . * Its slope of 2 could be due to either a vertical stretch or horizontal compression. |

**Objective 2:** In this section, you will create a table or equation to use as a model of a given function graph.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* [Recall](#Bookmark6) the rules for the different types of *transformations*.
* To create an equation of a transformed function:
  + Identify the parent function.
  + Examine the graph to determine which key features of the graph have changed compared to the parent function.
  + Make a table of corresponding *x-*values for each *y-*value of the parent and transformed functions.

|  |  |
| --- | --- |
| **Example:** Use the image to think about how the graph of the parent function has been transformed. | |
| **Step 1:** Identify the parent function. | The function graphed is a transformed quadratic function from the parent function . |
| **Step 2:** Examine the graph to determine which key features of the graph have changed compared to the parent function. | It appears to have been translated down four units, since the *y-*intercept is now located at (0, -4).  In addition, the graph appears to have been vertically stretched, as it seems narrower than a typical quadratic function. |
| **Step 3:** Compare the tables that represent the parent function and the transformed function. The points of the parent function can be found by graphing the parent function. | |  |  |  |  | | --- | --- | --- | --- | | ***Parent Function*** | | ***Transformed Function*** | | | *x* | *y* | *x* | *y* | | -2 | 4 | -2 | 4 | | -1 | 1 | -1 | -2 | | 0 | 0 | 0 | -4 | | 1 | 1 | 1 | -2 | | 2 | 4 | 2 | 4 | |
| **Step 4:** Follow the order of operations to determine how the transformed function will be written. | The function will be written as .   * The transformed function has a y-intercept that has been moved down 4 units, the value of is –4.   Determine by using the numbers in the table to help solve for .   * Use the point (-2, 4). * This shows that the parent function has been vertically stretched by a factor of 2. |
| **Step 4:** Write the transformed function. | The final translated function can be written as and illustrates a vertical stretch by a factor of 2 and a vertical shift down 4 units. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Use the image to answer the question.  A V shaped line with arrows at both ends is graphed on a coordinate plane labeled in increments of 1. The x-axis ranges from negative 9 to 9, and the y-axis from negative 11 to 11. Seven closed points are labeled.  Interpret the key features of the graph and identify the type of transformation that occurred. What type of transformation occurred from the parent function?  Option #1: An absolute value function has undergone a vertical translation.  Option #2: An absolute value function has undergone a horizontal translation.  Option #3: A quadratic function has undergone a vertical translation.  Option #4: A quadratic function has undergone a horizontal translation. | 2 |
| P 2 | Use the image to answer the question.  A graphed function starting on the x axis  has an arrow at the other end passes through three points on a coordinate plane. The x-axis ranges from negative 9 to 9 in unit increments and the y-axis ranges from negative 11 to 11 in unit increments.  The function  was transformed to create the function in the graph. Interpret the key features of the graph. What type of transformation occurred from the parent function?  Option #1: vertical translation  Option #2: horizontal translation  Option #3: vertical reflection  Option #4: horizontal reflection | 2 |
| P 3 | Use the image to answer the question.  Five points and a parabola are graphed on a coordinate plane. The x-axis ranges from negative 3 to 3 in increments of 1. The y-axis ranges from negative 1 to 9 in increments of 1.  Which equation models the transformation of the graphed function from the original function ?  Option #1:  Option #2:  Option #3: | 1 |
| P 4 | Use the image to answer the question.  Five points and one line are graphed on a coordinate plane. The x-axis ranges from negative 4 to 4 in increments of 1. The y-axis ranges from negative 10 to 10 in increments of 1.  What value of*k* would create the function  on the graph if it is a transformed version of the function ?  k = \_\_\_\_ | 3 |
| P 5 | Use the graph to answer the question.  A parabola is drawn in a coordinate plane where the axes range from negative 10 to 10, both by 1-unit increments.  The graph of a transformed function is shown. Enter the number of the option that states the parent function and the equation of the transformed function.  Option 1: The parent function is . The transformed function is  .  Option 2: The parent function is . The transformed function is  .  Option 1: The parent function is  . The transformed function is . | 1 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Use the image to answer the question.   An upward open parabola passes through 5 plotted points on a coordinate plane with x-axis ranging from negative 12 to 12 in increments of 2 and y-axis ranging from negative 6 to 18 in increments of 2.  Interpret the key features of the graph. What type of transformation occurred from the parent function? | This is a vertical stretch of the parent function due to the changes in the *y*-coordinates of each point. |
| Q 2 | Use the image to answer the question.  A coordinate plane's x-axis ranges from negative 12 to 12 and its y-axis ranges from negative 18 to 18, both by 2-unit increments. A curve of the function f left parenthesis x right parenthesis is plotted.  Interpret the key features of the graph to determine the transformation that occurred from its original function . | This graph represents a shift to the left due to the change in the *x*-intercept from the parent function. |
| Q 3 | Use the image to answer the question.  A parabola that opens upward and seven labeled points on the parabola are plotted on a coordinate plane. The parabola has bidirectional arrows and is labeled f left parenthesis x right parenthesis equals x squared baseline minus 3.  Which key feature helps to clearly illustrate that this graph has been moved down 3 units from the parent function ? | the *y*-intercept |
| Q 4 | Use the image to answer the question.  A parabola is plotted in a coordinate plane. The x-axis ranges from negative 4 to 4 and the y-axis ranges from negative 6 to 6 in one-unit increments.  Create an equation that models the transformation of  to the function in the graph. |  |
| Q 5 | Use the graph to answer the question.  A parabola is drawn on a coordinate plane. Both axes range from negative 5 to 5 in one-unit increments.  The graph of a transformed function is shown. Determine the parent function and the equation of the transformed function. | The parent function is . The equation of the transformed function is . |