Algebra 1

**Quadratic Equations**

**Unit Summary:** Quadratic equations help solve many types of real-world problems. In this unit, quadratic equations will be solved in various methods such as factoring and grouping. By the end of the unit, you will understand the relationship between exponents and roots to understand and solve quadratic equations.

[**GeoGebra Activities**](https://www.geogebra.org/search/Quadratic%20Equations)

**Lesson 2 – Solution Sets of Quadratic Equations**

**Key Words:**

* **equation** – a usually formal statement of the equality or equivalence of mathematical or logical expressions; a statement that two expressions are equivalent (have equal value)
* **evaluate** – to determine or fix the value of
* **quadratic equation** – any equation containing one term in which the unknown is squared and no term in which the unknown is raised to a higher power
* **satisfy** – to make true by fulfilling a condition
* **solution** – a value (number), that when substituted in for the variable of an equation, makes a true statement
* **solution set** – the set of values that satisfy an equation
* **squared** – raised to the second power, that is, multiplied by itself
* **substitute** – to put or use in the place of another

**Formulas:**

* Standard Form of a Quadratic Equation:

**Objective 1:** In this section, you willdetermine whether given variable values make a quadratic equation true or false.

*Mathematical Practice Standard: Attend to precision.*

**Big Ideas**:

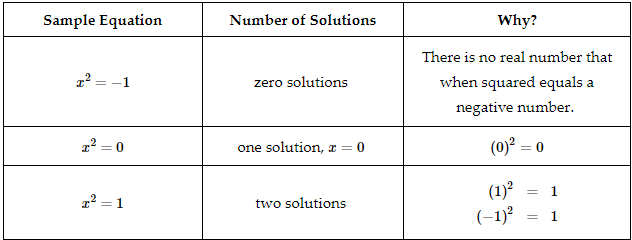
* Previously, you have solved linear equations. This unit will focus on *quadratic equations*.
* A *quadratic equation* is one of degree 2, meaning that of each term, the highest degree is 2, or the 2nd power.
  + Standard form of a *quadratic equation*:
* Notice in the following examples that the variable, *x*, is *squared*, making it a *quadratic equation*.
  + For example:
* Another common form for *quadratic equations* is called factored form.
  + One side of the equation contains factors that in turn contain the variable.
  + For example:
* By using *substitution*, you can determine whether a given value is a solution to a quadratic equation. Follow these steps:
  + 1. *Substitute* the value in for every occurrence of the variable in the equation.
  + 2. Evaluate or simplify to determine whether the equation is a true statement.
    - If true: the value is a solution
    - If false: the value is NOT a solution

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| **Example:** Given the quadratic equation , complete the table by testing each of the given values to determine whether it is a solution. Identify which of these values is in the solution set, if any. | | |
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| The values and are solutions, they are in the solution set. | | |

**Objective 2:** In this section, you will show that a quadratic equation can have zero, one, or two solutions.

*Mathematical Practice Standard: Attend to precision.*

**Big Ideas:**

* Recall that a *solution* is any value that, when *substituted* in for a variable, makes a true statement.
* Every quadratic equation will have a *solution set* of zero, one, or two solutions.
  + 
* Working backwards to rewrite the equation and isolate a variable in a *quadratic equation* will reveal how many *solutions* it has.
  + Use order of operations (PEMDAS) to work backwards until:
    - One side is only a squared variable expression (ie. )
    - The other side is only a numeric expression (ie. )
* Then, use the following guidelines to determine the number of *solutions*.
  + If the numeric expression is **greater than zero, there are two solutions**.
  + If the numeric expression is **equal to zero, there is one solution**.
  + If the numeric expression is **less than zero, there are no solutions**.

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| **Example:** Determine the number of solutions the following quadratic equation will have. | |
| **Step 1:** Rewrite the equation by working backwards using PEMDAS. | The last operation performed on the side with the variable is to subtract 5.  Undo the operation: |
| **Step 2:** Continue to rewrite the equation by working backwards using PEMDAS. | Now we have the expression on the left being multiplied by 2.  Undo the operation: |
| **Step 3:** Determine the number of solutions. | The equation is now in the form where the left side is a variable squared expression, and the right side is a numeric expression.  The numeric expression i s 100.  If the numeric expression is **greater than zero, there are two solutions**. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Determine whether the values 3 and 4 are solutions to the quadratic equation , and then select the correct answer from the following options.  Option #1: Only 𝑥=3 is a solution.  Option #2: Only 𝑥=4 is a solution.  Option #3: Both 𝑥=3 and =4 are solutions.  Option #4: Neither 𝑥=3 nor 𝑥=4 is a solution. | 3 |
| P 2 | *Use the table to answer the question.*   |  |  |  |  | | --- | --- | --- | --- | | *x* | **Substituted** | **Evaluate** | **True Statement?** | | −23 |  |  |  | | 0 |  |  |  | | 12 |  |  |  |   For the quadratic equation , complete the table by testing each of the given values to determine whether it is a solution. Identify which one of these values is in the solution set.  Only x=\_\_\_\_ is in the solution set. | -23 |
| P 3 | *Use the table to answer the question.*   |  |  |  |  | | --- | --- | --- | --- | | *x* | **Substituted** | **Evaluate** | **True Statement?** | |  |  |  |  | | 7 |  |  |  | |  |  |  |  | |  |  |  |  | |  |  |  |  |   For the quadratic equation , complete the table by testing each of the given values to determine whether it is a solution. Identify the two solutions to the quadratic equation.  The smaller of the two solutions is x=\_\_\_\_. The larger of the two solutions is x=\_\_\_\_. |  |
| P 4 | How many solutions would the equation  have? You do not need to solve for x.  The equation would have \_\_\_ solution(s). | 0 |
| P 5 | Assuming an equation with one side as a squared variable expression and the other side as a numeric expression, which of the following statements is correct?  Statement #1: If the numeric expression is greater than zero, there are two solutions.  Statement #2: If the numeric expression is greater than zero, there is one solution.  Statement #3: If the numeric expression is greater than zero, there are no solutions. | 1 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Determine whether the values −1 and  are solutions to the quadratic equation . | Both and are solutions. |
| Q 2 | Use the table to answer the question.   |  |  |  |  | | --- | --- | --- | --- | | x | **Substituted** | **Evaluate** | **True Statement?** | | 12 |  |  |  | | 24 |  |  |  | | 36 |  |  |  | | 48 |  |  |  |   When set off, a certain firework follows the path of the quadratic function , where:   * h= the height of the firework in feet. * x= the horizontal distance it travels in feet.   To determine how far the firework will travel before reaching the ground, determine which value of x in table is a solution to the equation 0. | 24 feet |
| Q 3 | Ariel is trying to determine if  is a solution to the quadratic equation . Which explanation demonstrates the correct reasoning? | Yes, is a solution because substituting it back into the equation results in the following: |
| Q 4 | Show how many solutions are in the solution set for the equation | two |
| Q 5 | Show how many solutions are in the solution set for . | two |

**Lesson 3 – Solving Simple Quadratic Equations**

**Key Words:**

* **equation** – a usually formal statement of the equality or equivalence of mathematical or logical expressions; a statement that two expressions are equivalent (have equal value)
* **inspection** – recognition of a familiar pattern leading to immediate solution of a mathematical problem
* inverse operation – an operation (such as subtraction) that undoes the effect of another operation (such as addition); also known as an inverse function
* **perfect square** – a number obtained from squaring an integer (multiply a number by itself)
* **quadratic equation** – any equation containing one term in which the unknown is squared and no term in which the unknown is raised to a higher power
* **radical sign** – the sign √ placed before an expression to denote that the square root is to be extracted or that the root marked by an index (as in √3 for the cube root) is to be extracted
* **satisfy** – to make true by fulfilling a condition
* **set** – a collection of elements and especially mathematical ones (such as numbers or points)
* **solution** – a set of values of the variables that satisfies an equation; for an equation with one variable, a value (number) that, when substituted in for the variable, makes a true statement
* **solution set** – the set of values that satisfy an equation
* **square root** – a factor of a number that when squared gives the number; for example, the square root of 9 is ±3
* **squared** – raised to the second power, that is, multiplied by itself

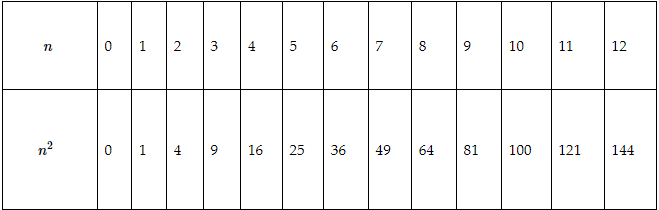
**Formulas:**

* Standard form of a Quadratic Equation:

**Objective 1:** In this section, you will solve quadratic equations of the form by inspection.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas**:

* Some *quadratic equations* with a certain structure, , can be solved in one step.
* You can use the concept of *inverse operations* or “working backward” to “undo” a *squared* expression.
* Recall that a *perfect square* is a number obtained from *squaring* an integer (multiplying a number by itself).
* If a *perfect square* appears in a *quadratic equation* with this structure, , you can solve it by *inspection*, with the positive or negative number that, when *squared*, equals that *perfect square*.
* Some common perfect squares:
  + 
* For example, has a solution set because:
  + We know there are two solutions because the numerical expression on the right is greater than zero.
  + We also know that 100 is a perfect square (refer to the chart).
  + and
* Sometimes, the solution set of a quadratic equation will be an empty set, meaning there are zero solutions. This can be written as {}.
  + For example, has a solution set of {} because the numerical expression on the right is less than zero.
* Not every *quadratic equation* with this structure will not have a *perfect square* on one side.

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| **Example:**  Since 55 is not a perfect square you can’t write an exact numeric solution. Follow these steps: | |
| **Step 1:** Find two perfect squares that the numeric expression falls between. | 55 is between the two perfect squares 49 and 64 |
| **Step 2:** Approximate the solution. | One solution is between 7 and 8.  The other solution is between –7 and –8. |

* You can also apply these concepts to rational expressions with *perfect squares*.

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| **Example:** Solve by inspection: | |
| **Step 1:** Evaluate the numerator, *n*. | The numerator can be 3 or –3. |
| **Step 2:** Evaluate the denominator, *d*. | The denominator can be 4 or –4. |
| **Step 3:** Put them together and state the solution. | For the solution set is . |

**Objective 2:** In this section, you will solve quadratic equations using square roots.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

* The *inverse operation* of *squaring* an expression is taking the positive or negative square root. Taking the *square root* effectively “undoes” the *square*.
* You will use this understanding to solve more complex *quadratic equations* by finding the *square root* of both sides of the equation.
* The general process for solving by square root is:

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| **Steps** | **Example #1** | **Example #2** |
| **Step 1:** If necessary, manipulate the equation using skills you have already learned so that it is in the form . |  |  |
| **Step 2:** Take the square root of both sides. If the constant is positive, take the positive and negative square root of both sides. |  | Note that 15 is not a perfect square, so it will stay in square root form for now. |
| **Step 3:** The previous step will yield two equations, one for the positive root and one for the negative root. State the equations. | Equation 1:  Equation 2: | Equation 1:  Equation 2: |
| **Step 4:** Solve the resulting linear equations to find the solution set. | Equation 1:  Equation 2: | Equation 1:  Equation 2: |
| **Step 5:** State the solution set. |  | or |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Solve  by inspection. There are two real solutions. Enter the lesser number first.  {\_\_\_,\_\_\_} | -6; 6 |
| P 2 | Solve  by inspection. There are two real solutions. Enter the lesser number first. Leave the answers in simplest fraction form.  {\_\_\_,\_\_\_} |  |
| P 3 | Solve the following quadratic equation using square roots: .  {\_\_\_, \_\_\_} | 3, -9 |
| P 4 | Solve the following quadratic equation using square roots: .  {\_\_\_, \_\_\_} | 13.14, -1.14 |
| P 5 | Solve the following quadratic equation using square roots. Round to the nearest hundredth if necessary: .  {\_\_\_, \_\_\_} | -8.06, -23.94 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Use inspection to solve the equation , then select the complete solution set below. If there are no real solutions, select “There are no real solutions.” | {-9, 9} |
| Q 2 | Use inspection to solve the equation , then select the correct solution set below. If there are no real solutions, select “There are no real solutions.” |  |
| Q 3 | Solve the following quadratic equation using square roots: . | {32, 4} |
| Q 4 | Solve the following quadratic equation using square roots: | {-2, -10} |
| Q 5 | Solve the following quadratic equation using square roots: . | {-19, -53} |

**Lesson 4 – The Zero Product Property**

**Key Words:**

* **factor** – any of the numbers or symbols in mathematics that when multiplied together form a product; to find the mathematical factors and especially the prime mathematical factors
* **factored form** – a form in which a quadratic equation is expressed as a product of two algebraic expressions
* **factored form of a quadratic equation** – a form in which a quadratic equation is expressed as a product of two algebraic expressions
* **product** – the number or expression resulting from the multiplying together two or more numbers or expressions
* **quadratic equation** – any equation containing one term in which the unknown is squared and no term in which the unknown is raised to a higher power
* **root** – a quantity taken an indicated number of times as an equal factor
* **solution** – a set of values of the variables that satisfies an equation; for an equation with one variable, a value (number) that, when substituted in for the variable, makes a true statement
* **solution set** – the set of values that satisfy an equation
* **solve** – to find a solution, explanation, or answer for
* **Zero Product Property** – a property stating that if the product of two expressions or quantities is equal to zero, then at least one of the expressions or quantities is equal to zero

**Formulas:**

* Zero Product Property: If then , , or both and are zero.
* Standard form of a Quadratic Equation:
* Factored Form of a Quadratic Equation:

**Objective 1:** In this section, you will explain how the Zero Product Property can be used to find solution sets of quadratic equations.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas**:

* Recall that quadratic equations can be written in *factored* form. When in factored form, you can use the *Zero Product Property* to solve the equation.
  + The factored form of a quadratic equation has a specific structure:
  + For example, , is the factored form of a quadratic equation.
  + We know this because when we distribute or “multiply out” the left side, you obtain .
* *Zero Product Property*: If a product equals zero, then at least one of the *factors* must equal zero.
  + - Set each factor to 0 separately and solve for the variable to obtain the solution set of the quadratic equation.

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| **Example:** What is the solution set of the equation ? | |
| **Step 1:** Use the Zero Product Property to write two equations. | If the equation equals zero, then one or both factors must be equal to zero. The only solutions to the equation are those that make a factor equal to zero. |
| **Step 2:** Solve each equation for the variable. | Equation 1:  Equation 2: |
| **Step 3:** State the solution set. | Any value of x that makes either one of these equations true is a solution to the original equation.  The solution set is . |

**Objective 2:** In this section, you will use the Zero Product Property to solve quadratic equations that are in factored form.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

* Recall the *Zero Product Property* that states if the product of factors equals zero, then at least one of the factors must equal zero.
* Recall the factored form of a quadratic equation:
  + A factor in factored quadratic equations does not always have the form of .
  + Other forms of factors include where *a* and *b* are constants.

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| **Example:** Solve the following quadratic equation using the Zero Product Property: | |
| **Step 1:** Confirm that the equation is in the correct structure for the Zero Product Property. | The side of the equation that contains the variable has three factors.  The equation is set to equal 0.  Yes, the Zero Product Property can be applied. |
| **Step 2:** Set each factor with a variable equal to zero. | Factor 1: 2  2 cannot equal zero so it is excluded.  Factor 2:  Factor 3: |
| **Step 3:** State the solution set. | The solution set is . |

* Using what you know about the *factors* of a *quadratic equation* will help you write a *quadratic equation* given just the *solution set*.
  + Use the steps for solving an equation in *factored form* (above example) and work backwards.

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| **Example:** Given the solution set of , write a quadratic equation in factored form. | |
| **Step 1:** Set each solution equal to *x*. |  |
| **Step 2:** Use inverse operations to rearrange the equation so that the variable and constant are on the left and the right side is 0. | Equation 1:  Equation 2: |
| **Step 3:** The resulting equations are also the quadratic equations factors. | One quadratic equation could be: |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Use the Zero Product Property to find the value of x that makes the following statement true: .  The value that makes the statement true is x=\_\_\_\_\_. | 3 |
| P 2 | Zavier, Boaz, and Sophie are trying to find the values of x that make the following quadratic function equal 0: . Each student came up with a different answer, which they explained in the options. Which student’s explanation is correct?  Option #1: Zavier says that only  will make the function equal 0, and he provided the given work: .  Option #2: Boaz says that the values  and  will both make the function equal 0, and he provided the given work: , and .  Option #3: Sophie says that only  will make the function equal 0, and she provided the given work: .  Option #\_\_\_\_ is the correct explanation. | 2 |
| P 3 | Use the table to answer the question.    It is only possible to use the Zero Product Property on one of the options in this table. Which option can the Zero Product Property be used on to find the zeros of the function? | 2 |
| P 4 | Solve the factored quadratic equation .  The smaller solution is 𝑥=\_\_\_\_\_, and the larger solution is 𝑥=\_\_\_\_\_. | -3; 5 |
| P 5 | To begin a football game, a kicker has to kick off. The football follows the path , where ℎ(𝑥) is the height of the football and x is the horizontal distance it has traveled in yards. Solve the equation to determine how far the ball will have traveled when it hits the ground.    The ball will have traveled \_\_\_\_ yards before hitting the ground. | 61 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | *Use the table to answer the question.*   |  | | | --- | --- | |  |  | |  |  |   While using the Zero Product Property to find the values of *x* that make the quadratic equation  equals 0, Oliver completed the work provided in the table. Is Oliver’s work accurate? | No, x-3 = 0 in the second row should be x+3 = 0. |
| Q 2 | Based on the Zero Product Property, which of the following statements must be true about the quadratic equation ? | At least one of the factors, (2x−3) or (x+2), must equal 0 for the entire function to equal 0. |
| Q 3 | The path of a soccer ball can be modeled by the equation , where h(x) is the height of the ball, and x is the horizontal distance the ball has traveled. To determine how far the ball has traveled when it hits the ground, Leon set the height equal to 0 and obtained the equation: . How can he use the Zero Product Property to find the horizontal distance the ball has traveled when the height of the soccer ball equals 0? | Solve the two equations: and |
| Q 4 | Solve the factored quadratic equation . | The solution set is . |
| Q 5 | A golfer hits a golf ball toward the hole 55 yards away. The ball follows a parabolic path given by the function , where  is the height of the golf ball and x is the horizontal distance it has traveled. Solve the equation to determine how far the ball will have traveled when it hits the ground. How far will the ball need to roll to make it to the hole? | The ball will hit the ground after it has traveled 45 yards. It will need to roll an additional 10 yards to reach the hole. |

**Lesson 5 – Solving Quadratic Equations Using Common Factors**

**Key Words:**

* **binomial** – an algebraic expression with two terms
* **coefficient** – a number used to multiply a variable
* **constant** – a quantity having a fixed value that does not change or vary, such as a number
* **factor** – any of the numbers or symbols in mathematics that when multiplied together form a product; to find the mathematical factors and especially the prime mathematical factors
* **factored form of a quadratic equation** – a form in which a quadratic equation is expressed as a product of two algebraic expressions
* **greatest common factor (GCF)** – the largest factor that can be divided from all terms of an expression
* **monomial** – a mathematical expression consisting of a single term
* **quadratic equation** – any equation containing one term in which the unknown is squared and no term in which the unknown is raised to a higher power
* **shared binomial** – an expression that repeats after the greatest common factor has been divided out
* **standard form of a quadratic equation** – a form in which a quadratic equation is written as , where
* **term** – is either a single number or variable, or numbers and variables multiplied together
* **Zero Product Property** – a property stating that if the product of two expressions or quantities is equal to zero, then at least one of the expressions or quantities is equal to zero

**Formulas:**

* Standard form of a Quadratic Equation:
* Factored Form of a Quadratic Equation:
* Zero Product Property: If then , , or both and are zero.
* Distributive Property:

**Objective 1:** In this section, you will solve quadratic equations by factoring out the greatest common factor (GCF).

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas**:

* Some *quadratic equations* that are not in *factored form* can be written in *factored form*, or, as the product of *factors*. This process of rewriting a *quadratic equation* is called *factoring*.
  + Once factored, you can solve using the [*Zero Product Property*](#Bookmark2).
* The first form of *factoring* that you will learn is by “factoring out” the *greatest common factor (GCF)*.
  + The GCF is the largest factor divided from all terms of an expression.
  + The equation must be in the form
* Practice finding the GCF of numbers by finding the prime factorization of each number and then multiply the common prime factors together.

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| **Example:** Find the GCF of 540 and 882 using prime factorization. | |
| **Step 1:** Use a factor tree to find the prime factors of 540. |  |
| **Step 2:** Use a factor tree to find the prime factors of 882. |  |
| **Step 3:** Identify the common factors. | The GCF contains the common factors – the factors in **both** factor trees (in yellow):   * 1 factor of 2 * 2 factors of 3 * No factors of 5 or 7 |
| **Step 4:** Multiply the common factors together to obtain the GCF. | The GCF is 18. |

* You can find the GCF of a variable raised to different powers by using an exponent. The GCF is the lowest exponent or lowest degree among all the terms.
* Use the following steps to factor a quadratic equation using the GCF:

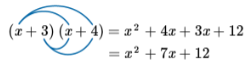
|  |  |
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| **Example:** Solve the quadratic equation by factoring out the GCF. | |
| **Step 1:** Write the equation in standard form . | The equation is already written in standard form: |
| **Step 2:** Find the GCF of all terms. | The GCF of and is:  The GCF of 12 and 18 is:  The polynomials GCF is . |
| **Step 3:** Rewrite each term as a product of the GCF and another factor. |  |
| **Step 4:** Apply the Distributive Property using the formula . | If , , and then the equation can be written as: |
| **Step 5:** Solve the factored form by applying the Zero Product Property. | The two factors are .  Set each equal to zero and solve for x.  Equation 1:  Equation 2: |
| **Step 6:** State the solution set. | The solution set is . |

**Objective 2:** In this section, you will solve quadratic equations by grouping.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

* Recall that to multiply two binomials together such as use the *Distributive Property*



* Recall that a quadratic equation is in the form .
* Factoring by grouping requires splitting the middle term of the quadratic equation into two terms, grouping the terms, then finding the greatest common factor (GCF).
* The exact steps are outlined with the following example:

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| **Example:** Solve by grouping. | |
| **Step 1:** Check if the terms have a common factor. If there is a common factor, divide the equation by the factor. | The terms of the given quadratic equation have no common factor except 1. Therefore, the equation will stay the same for the remaining steps. |
| **Step 2:** Find . |  |
| **Step 3:** Find the factors of whose sum is (the coefficient of the middle term.) | The factors of could be any two numbers that multiply to 12. For them to be factors of the given quadratic equation, they must also add to (the coefficient of the middle term.)    The only factors that multiply to 12 and add to 7 are 3 and 4. |
| **Step 4:** Split the middle term into the sum of the two new terms found in step 3. | becomes |
| **Step 5:** Separate the first two terms from the last two terms using addition, so that you have two groups of terms. | becomes |
| **Step 6:** Find the GCF for each group and factor it out. | * The terms of the first group are and , which has a common factor of .   + Factoring out the term becomes * The terms of the second group are and , which has a common factor of 4.   + Factoring 4 out the term becomes * Put these together: |
| **Step 7:** Factor out the shared binomial. | Both terms share the binomialwhich can be factored out of each term.  becomes |
| **Step 8:** Use the [Zero Product Property](#Bookmark2) to solve the equation. | Equation 1:  Equation 2: |
| **Step 9:** State the solution set. | The solutions to the quadratic equation are –3 and –4. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Solve the following quadratic equation by factoring out the greatest common factor (GCF): .  The smaller solution is 𝑥=\_\_\_\_, and the larger solution is 𝑥=\_\_\_\_. | 0; 7 |
| P 2 | Rewrite the following quadratic equation in standard form and then solve by factoring out the GCF: .  The smaller solution is 𝑥=\_\_\_\_, and the larger solution is 𝑥=\_\_\_\_. | 0; |
| P 3 | During a water balloon fight, Louisa uses a slingshot to fire a water balloon over the fortified wall of the opponent. When fired from the slingshot, the balloon will follow the path , where ℎ(𝑥) is the height of the balloon in feet, and x is the horizontal distance it will travel. To determine how far the balloon will travel before it reaches the ground, Louisa sets the height equal to zero and solves the quadratic equation 0. How far will the balloon travel before it reaches the ground?  The balloon will travel \_\_\_\_ feet before it reaches the ground. | 12 |
| P 4 | Karim is solving the quadratic equation  by grouping. His work is shown below. Determine the first step at which Karim makes an error.  Step 1:  Step 2:  Step 3:  Step 4:  Step 5:  or  𝑥=−8 or 𝑥=−3  Karim's first mistake occurs at Step \_\_\_\_. | 3 |
| P 5 | One of the steps needed to solve a quadratic equation by grouping is to split the middle term as a sum of two terms. Given the equation , which two numbers would be needed to complete this step? Enter the lesser value first. | -7; -6 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Solve the quadratic equation  by factoring out the GCF. | The solutions are x=−15 and x=0. |
| Q 2 | Which of the following tables shows the correct steps to factor out the GCF and solve the quadratic equation ? |  |
| Q 3 | As Isla prepares to set off fireworks, she wants to ensure the spectators are a safe distance away. When shot in the air, one firework will follow the path , where h(x) is the height of the firework, and x is the horizontal distance it travels in feet. To determine how far the firework will travel before it reaches the ground, Isla sets the height equal to zero, and solves the following quadratic equation: 0. How far will the firework travel before it reaches the ground? | 15 feet |
| Q 4 | Hyung-eun is solving the quadratic equation  by grouping. Her work is shown below. At which step does Hyung-eun first make an error?  Step1:  Step 2:  Step 3:  Step 4:  Step 5: x+6=0 or x−2=0  x=−6 or x=2 | Step 4 |
| Q 5 | Which of the following quadratic equations can be solved by grouping? |  |

**Lesson 6 – Solving Quadratic Equations Using Patterns**

**Key Words:**

* **binomial** – an algebraic expression with two terms
* difference of two squares – the square of a number or expression minus the square of another number or expression; follows the form
* **factored form of a quadratic equation** – a form in which a quadratic equation is expressed as a product of two algebraic expressions
* **linear equation** – an equation containing a variable raised to the power of 1; an equation in the form where
* **perfect square** – a number obtained from squaring an integer (multiply a number by itself)
* **perfect square trinomial** – a trinomial that can be written as a square of a binomial
* **quadratic trinomial** – a trinomial that can be written in the form , where
* **standard form of a quadratic equation** – a form in which a quadratic equation is written as , where
* **sum-product pattern** – a factoring method for a quadratic expression written in standard form with a=1, in which factors need to be found for the constant term, c, whose sum is the coefficient of the middle term, b
* **trinomial** – an algebraic expression with three terms

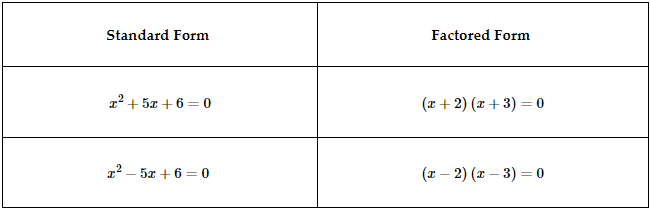
**Formulas:**

* Standard Form of a Quadratic Equation:
* Perfect Square Trinomial:
  + 
* Zero Product Property: If then , , or both and are zero.
* Difference of Two Squares:

**Objective 1:** In this section, you will solve quadratic equations by factoring using the sum-product pattern.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas**:

* Recall that the standard form of a quadratic equation is where .
* [Recall](#Bookmark1) how to *factor* a *quadratic equation* from *standard form* into its *factored form* using grouping.
* Recall that once a *quadratic* is expressed in its *factored form*, you can use the [*Zero Product Property*](#Bookmark2) to find the *solutions*.
  + 
* You can also use the *sum-product pattern* to express a *standard form quadratic equation* in its *factored form.* This method only works when the leading term of the quadratic equation is *.*
  + *Sum-Product Pattern*: The coefficient to the middle term, , is equal to the sum of the factors of the constant term, .
* Here are the steps you can follow to solve *quadratic equations* by *factoring* using the *sum-product pattern*:

|  |  |
| --- | --- |
| **Example:** Solve by factoring. | |
| **Step 1:** Move all the terms to the left side of the equation, and make sure the right side of the equation is zero. |  |
| **Step 2:** Find the factors of the constant term, , whose sum is also the coefficient of the middle term, . | Standard form of a quadratic:  The equation to solve:  Following the pattern of the standard form of a quadratic, and .  List the factors of . Identify the two factors that sum to 6.   |  |  |  | | --- | --- | --- | | **Factors** | | **Sum** | | 1 | 8 |  | | -1 | -8 |  | | 2 | 4 |  | | -2 | -4 |  |   The factors 2 and 4 are factors of that sum to |
| **Step 3:** Write the quadratic equation in its factored form. | Use factors 2 and 4.  becomes |
| **Step 4:** Use the Zero Product Property to solve the equation. | Set each factor equal to zero and solve for *x*. |
| **Step 5:** State the solutions. | The solutions of are and . |

**Objective 2:** In this section, you will solve quadratic equations by factoring using the perfect square trinomial pattern.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

* A *trinomial* that can be written in the form , where , is called a *quadratic trinomial*.
* A *quadratic trinomial* can be expressed as a product of two *binomials* through factoring.
  + For example, can be written as .
* A *perfect square trinomial* is a trinomial that can be written as the square of a binomial.
* For example, when the following *perfect square trinomials* are rewritten in factored form, the binomial factors repeat, or are the same, which can then be written as the square of a binomial.

|  |  |  |
| --- | --- | --- |
| **Standard Form** | **Factored Form** | **Square of a Binomial** |
|  |  |  |
|  |  |  |
|  |  |  |

* *Perfect square trinomials* follow a pattern that can be used to factor the trinomial.
  + 
  + The first and last terms of the trinomial ( and ) are positive and *perfect squares*.
    - [Recall perfect squares](#Bookmark3).
  + The middle term ( or ) is twice the product of and .
  + Notice there are two different patterns depending on if the middle term is positive or negative.

|  |  |
| --- | --- |
| **Example:** Solve . | |
| **Step 1:** Make sure all the terms are on the left side of the equation and that the right side is zero. |  |
| **Step 2:** Identify any common factors among the terms to further simplify. | All terms have a common factor of 2. Divide both sides by 2. |
| **Step 3:** Check if the expression on the left side follows the pattern for a perfect square trinomial. | The left side of the equation is:   * The first and last terms are perfect squares ( and ). * The middle term , is twice the product of the of and . * This fits the pattern for a perfect square trinomial: |
| **Step 4:** Rewrite the expression in the form . |  |
| **Step 5:** Use the pattern to express the left side of the equation as a binomial squared. |  |
| **Step 6:** Rewrite the original equation with the left side expressed as the squared binomial. |  |
| **Step 7**: Take the square root of both sides. |  |
| **Step 8:** Solve the resulting linear equation. |  |
| **Step 9:** State the solution. | The solution of is . |

**Objective 3:** In this section, you will solve quadratic equations by factoring using the difference of squares pattern.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

* The expression is called a *difference of two squares* and it can also be used as a factoring pattern similar to[*perfect square trinomials*](#Bookmark4)*.* 
  + If there are only two terms that are [*perfect squares*](#Bookmark3), and they are separated by a minus sign, it can be written as a *difference of two squares.*

|  |  |
| --- | --- |
| **Example:** Solve . | |
| **Step 1:** Make sure all terms are on the left side of the equation and that the right side is zero. |  |
| **Step 2:** Factor the left side of the equation using the difference of two squares pattern. | Difference of Two Squares:  The terms on the left side of the equation are perfect squares and are separated by a minus sign.  so  so |
| **Step 3:** Use the Zero Product Property to solve. |  |
| **Step 4:** State the solutions. | The solutions of are and . |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Supply the numbers to write  in its factored form. Enter the lesser number first.  (𝑥+\_\_\_)𝑥+ \_\_\_)=0 | 2; 8 |
| P 2 | Supply the number so that the quadratic equation below has 6 and 8 as its solutions. | 48 |
| P 3 | Solve the following quadratic equation using the perfect square trinomial pattern: .  *x* = \_\_\_\_\_ | -5 |
| P 4 | Solve the following quadratic equation using the perfect square trinomial pattern: .  *x* = \_\_\_\_\_ | -27 |
| P 5 | What are the solutions to ? Enter the lesser number first.  *x* = \_\_\_\_\_, *x* = \_\_\_\_\_ | -11; 11 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | The quadratic equation  can be solved by factoring. Which of the following is the factored form? |  |
| Q 2 | Which of the following equations has exactly the same solutions as ? |  |
| Q 3 | What should be the value of b in  so that its solutions are −11 and 5? | 6 |
| Q 4 | Solve the following quadratic equation using the perfect square trinomial pattern: . | x = 8 |
| Q 5 | Gabriel was asked to solve  by factoring using the difference of squares pattern. What is/are the correct solution(s)? | x=6, x=−6 |

**Lesson 7 – Completing the Square & the Quadratic Formula**

**Key Words:**

* **completing the square** – the process of making an expression a perfect square trinomial
* **formula** – an equation that describes how two or more quantities are related
* **literal equation** – an equation that contains two or more variables
* **perfect square trinomial** – a trinomial that can be written as a square of a binomial
* **quadratic equation** – any equation containing one term in which the unknown is squared and no term in which the unknown is raised to a higher power
* **quadratic formula** – a formula that gives the solutions of a quadratic equation expressed in standard form, , where and that is written in the form

**Formulas:**

* Perfect Square Trinomial:
  + 
* Completing the Square:
* Quadratic Formula:
* Standard Form Quadratic:
* Discriminant of a Parabola:

**Objective 1:** In this section, you will complete the square to solve quadratic equations.

*Mathematical Practice Standard: Look for and express regularity in repeated reasoning.*

**Big Ideas**:

* *Completing the square* is the process of making an expression a *perfect square trinomial*.
* This process can be used to solve any quadratic equation where .
  + The first step in *completing the square* is to move the constant term, , to the right side of the equation:
  + Then you will divide the coefficient of the middle term, , by 2, then squaring the quotient:
  + Then add the number to both sides of the equation:
  + From there, you can continue with factoring.
* The following example outlines the steps for completing the square:

|  |  |
| --- | --- |
| **Example:** Solve by completing the square. | |
| **Step 1:** Move the constant term to the right side of the equation. |  |
| **Step 2:** Complete the square by dividing the coefficient of the middle term, , by 2, then squaring the quotient. | The coefficient of the middle term, , is –10. |
| **Step 3:** Add the number found in Step 2 to both sides of the equation. |  |
| **Step 4:** Factor the left side using the [perfect square trinomial pattern](#Bookmark4), and simplify the right side. | The left side of the equation follows the perfect square trinomial pattern: .  and  The left side of the equation can be written as .  Rewrite the equation: |
| **Step 5:** Take the square root of both sides. |  |
| **Step 6:** Create two equations for each sign on the right side and solve for *x*. | Equation 1: |
|  | Equation 2: |
| **Step 7:** State the solutions. | The solutions are and . |

**Objective 2:** In this section, you will derive the quadratic formula from the process of completing the square.

*Mathematical Practice Standard: Construct viable arguments and critique the reasoning of others.*

**Big Ideas:**

* Recall in previous lessons when you learned how to rearrange *literal equations* to be written in terms of different quantities of interest.
  + To solve a formula is to isolate any variable of interest on one side by applying inverse operations and properties of equality.
* [Recall](#Bookmark5) how to solve a quadratic equation by *completing the square*.
* When is isolated in the standard form of a quadratic equation by *completing the square* and rearranging, the result is the *quadratic formula*:

|  |
| --- |
| **The Quadratic Formula**  where , , and are from the standard form of a quadratic equation |

**Objective 3:** In this section, you will use the quadratic formula to solve quadratic equations.

*Mathematical Practice Standard: Attend to precision.*

**Big Ideas:**

* Recall the *quadratic formula*:
* The *quadratic formula* can be used to solve quadratic equations in standard form ().
* After identifying the values of , , and , you can substitute these values into the *quadratic formula* to solve for .

|  |  |
| --- | --- |
| **Example:** Solve . | |
| **Step 1:** Arrange the equation into the standard form of a quadratic equation so that it is equal to zero. | Standard Form: |
| **Step 2:** Identify the values of , , and . | Standard Form:  Equation to solve:  and . |
| **Step 3:** Substitute the values for , , and into the quadratic formula. |  |
| **Step 4:** Simplify the equation step-by-step. Pay close attention to the order of operations and signed number rules. |  |
| **Step 5:** Find the solutions. | Find the solution for the addition case and find the solution for the subtraction case. |
| **Step 6:** State the solutions. | The solutions for are and . |

* In the quadratic formula, , is known as the discriminant of a parabola.
* It determines how many solutions a quadratic equation has without finding them.
  + If is positive, then there are two solutions.
  + If is zero, then there is one solution.
  + If is negative, then there is no real number solution.

|  |  |
| --- | --- |
| **Example:** How many solutions does the quadratic equation have? | |
| **Step 1:** Identify the values for , , and . | Standard Form:  Equation: |
| **Step 2:** Substitute the values into the formula for the discriminant. |  |
| **Step 3:** Determine the number of solutions. | The discriminant is a negative number. Therefore, there is no real number solution. |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Complete the square to identify the two values that solve the following quadratic equation: .  𝑥=\_\_\_\_\_ | 1, -9 |
| P 2 | Complete the square to solve the following quadratic equation: . Is the following correct: ?  Enter 1 for Yes.  Enter 2 for No. | 1 |
| P 3 | Using the quadratic formula, find the solution to | -3 |
| P 4 | Solve using the quadratic formula. Round the solution(s) to the nearest hundredths. Enter the smaller solution first.  or \_\_\_\_\_ | -0.28; 1.78 |
| P 5 | How many real solutions does the following quadratic equation have?  \_\_\_\_\_ solution(s) | 1 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Complete the square to solve the following quadratic equation: . | x=2, x=−4 |
| Q 2 | Complete the square to solve the following quadratic equation: . |  |
| Q 3 | Using the quadratic formula, find the solution: . | x=8, x=−2 |
| Q 4 | Which of the following is a solution to the quadratic equation ? Assume that the solution has been rounded to the nearest hundredth, if applicable. | x = -0.89 |
| Q 5 | How many real solutions does the following quadratic equation have? | No real solutions |

**Lesson 8 – Graphs of Quadratic Equations**

**Key Words:**

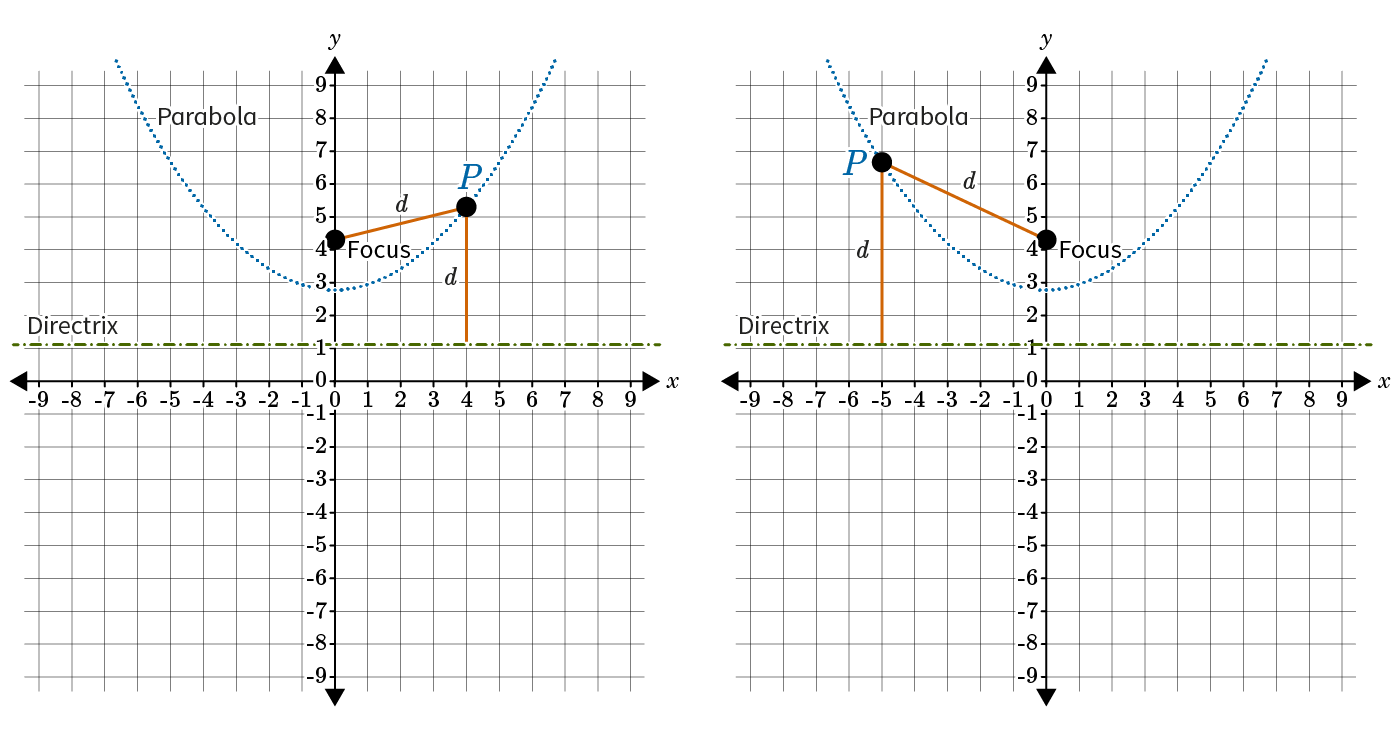
* **axis of symmetry of a parabola** – the straight line that divides a parabola into two identical parts
* **parabola** – a curve where any point is equidistant from a fixed point (the focus) and a fixed straight line (the directrix); the focus may not lie on the directrix
* **quadratic equation** – any equation containing one term in which the unknown is squared and no term in which the unknown is raised to a higher power
* **vertex of a parabola** – the highest or lowest point of a parabola that crosses its axis of symmetry
* ***y*-intercept of a parabola** – the point where a parabola intersects the *y*-axis
* **zeros of a parabola** – the points where a parabola intersects the *x*-axis; the *x*-values that make the quadratic equation equal to zero

**Objective 1:** In this section, you will generate points using quadratic equations to create corresponding graphs, called parabolas.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* When you plot points using a *quadratic equation* the graph is a *parabola*.
* A parabola is a curve where any point is the same distance from:
  + a fixed point (the focus) and
  + a fixed straight line (the directrix)
* In the graphs below, notice that point *P* on the *parabola* is the same distance, *d*, from the focus as it is from the directrix. This same relationship holds true for ALL points on a *parabola*.



* You can interpret certain features of a *parabola* by inspecting its *quadratic equation*.

|  |  |
| --- | --- |
| Upward U-Shape | Downward U-Shape |
| When the value of in a quadratic equation is **positive**:   * The parabola will be an **upward** U-shape. * The parabola will have a **minimum**, which is the **lowest** *y*-value. | When the value of in a quadratic equation is **negative**:   * The parabola will be a **downward** U-shape. * The parabola will have a **maximum**, which is the **highest** *y*-value. |
| For example: | For example: |

* To graph a *parabola* from a *quadratic equation*, you must generate a set of points.

|  |  |
| --- | --- |
| **Example:** Generate points using the quadratic equation to create the corresponding parabola. | |
| **Step 1:** Make a table of values (points) for the parabola by selecting a set of symmetrical values for . Substitute these values into the equation to determine the *y*-value.  You can also generate a table of values in GeoGebra by following these [instructions](https://www.geogebra.org/calculator/xyygsrgp)! | |  |  |  |  | | --- | --- | --- | --- | | *x* | Substitute and Solve | *y* | Coordinate Point | | -2 |  | 5 |  | | -1 |  | 0 |  | | 0 |  | -3 |  | | 1 |  | -4 |  | | 2 |  | -3 |  | | 3 |  | 0 |  | | 4 |  | 5 |  | |
| **Step 2:** Plot the points on the coordinate plane. |  |
| **Step 3:** Connect the points to show the parabola. |  |

**Objective 2:** In this section, you will identify the vertex, axis of symmetry, zeros, and *y*-intercepts of graphs of quadratic equations.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* From generating a set of points and graphing a parabola, you will be able to identify key features and graph the *parabola*.

|  |  |
| --- | --- |
| **Axis of Symmetry**  A straight line that divides a parabola into two identical parts.  (blue dashed line) | * Axis of Symmetry: * Vertex: (1, -4) * Zeros: (-1,0) and (3,0) * y-intercept: (0, -3) |
| **Vertex**  The highest or lowest point of the parabola that crosses its axis of symmetry. |
| **Zeros**  The points where the parabola intersects the *x*-axis; they are also the *x*-values that make the quadratic equation equal to zero.  The *y*-coordinates of the zeros will be zero. |
| ***y*-intercept**  The point where the parabola intersects the *y*-axis. The *x*-coordinate of the y-intercept is zero.  The y-intercept may also be identified by the vlaue in the quadratic equation . |

* There are three cases for *zeros of parabolas*. There can be none, one, or two zeros depending on the graph.

|  |  |  |
| --- | --- | --- |
| **No Zeros** – the parabola does not intersect the x-axis | **One Zero** – the parabola touches the x-axis at one point | **Two Zeros** – the parabola crosses the x-axis at two points |
|  |  |  |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Given the quadratic equation , solve for the y-coordinate of the parabola when 𝑥=−3.  (−3, \_\_\_\_\_) | 30 |
| P 2 | Given this table of values, complete the table for the quadratic equation  to find the values of the high or low point of the parabola.    The high or low point of this parabola is (\_\_\_\_, \_\_\_\_). | 1; 6 |
| P 3 | Use the image to answer the question.  Identify the vertex and axis of symmetry of the parabola.  axis of symmetry: 𝑥=\_\_\_\_\_  vertex: (\_\_\_\_) | -10; -10, 0 |
| P 4 | Use the image to answer the question.    Identify the vertex and axis of symmetry of the parabola.  axis of symmetry: 𝑥=\_\_\_\_\_  vertex: (\_\_\_\_) | -1; -1, 16 |
| P 5 | Use the image to answer the question.  Identify the y-intercept and vertex of the parabola.  vertex: (\_\_\_)  y-intercept: (\_\_\_) | 1, -3; 0, -4 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Which of the following is a point you can generate to create the parabola for the quadratic equation ? | (-3, -55) |
| Q 2 | Given the table of values, complete the table for the quadratic equation  to find the values of the high or low point of the parabola.    Which of the following is correct? | (0.25, 8.375) is the high point of the parabola |
| Q 3 | Use the image to answer the question.  Identify one of the zeros (or x-intercepts) on the graph. | (4, 0) |
| Q 4 | Use the image to answer the question.  Identify the vertex on the graph. | (-11, -1) |
| Q 5 | Use the image to answer the question.  Determine which of the following statements is true about the parabola. | The *y*-intercept and the vertex share the same point. |

**Lesson 9 – Features of Quadratic Equations**

**Key Words:**

* **axis of symmetry of a parabola** – the straight line that divides a parabola into two identical parts
* **completing the square** – the process of making an expression a perfect square trinomial
* **perfect square trinomial** – a trinomial that can be written as a square of a binomial
* **vertex of a parabola** – the highest or lowest point of a parabola that crosses its axis of symmetry
* ***x*-intercepts of a parabola** – the points where a parabola intersects the x-axis; the x-values that make the quadratic equation equal to zero; also call zeroes
* ***y*-intercept of a parabola** – the point where a parabola intersects the *y*-axis
* **zeros of a parabola** – the points where a parabola intersects the *x*-axis; the *x*-values that make the quadratic equation equal to zero
* **zeros of a quadratic equation** – the *x*-values that make the quadratic equation equal to zero

**Formulas:**

* Vertex Form:
  + Axis of Symmetry:
  + Vertex:
* Perfect Square Trinomial:
  + 
* Quadratic Formula:

**Objective 1:** In this section, you will use multiple methods to locate zeros of quadratic equations.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas**:

* The *zeros of a quadratic equation* are the *x*-values that make the quadratic equation equal to zero. They are found in many ways.

|  |  |
| --- | --- |
| **Method** | **Example:** |
| **Factoring**  Recall the different methods for factoring like grouping and [completing the square](#Bookmark5). | * Set the equation equal to zero. * Factor the equation. * Set each factor equal to zero and solve.   The zeros are and or (-5,0) and (3,0). |
| **Table**  Use a graphing calculator or other technology to generate a table of values.  Search for the ordered pairs where the *y*-values equal zero. These are the *x*-intercepts, or zeros. | |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | ***x*** | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | | ***y*** | 0 | -7 | -12 | -15 | -16 | -15 | 12 | 7 | 0 |   The zeros are and . |
| **Graph**  The graph of a quadratic equation is a parabola.  The *zeros of a parabola* are the points where a parabola intersects the *x*-axis. They are the *x*-intercepts. | The *x*-intercepts (-5,0) and (3,0) are the zeros. |
| **Quadratic Formula**  The zeros may be found using the [quadratic formula](#Bookmark6). You can use this formula on any quadratic equation, but it is most helpful when the equation is not factorable. | * Set the equation equal to zero. * Identify the values of , , and and substitute them into the quadratic formula. * Simplify. * Find the solutions.   The solutions/zeros are and . |

**Objective 2:** In this section, you will complete the square to locate the vertex and axis of symmetry of quadratic equations.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas:**

* *Completing the square* converts a *quadratic equation* in standard form, , into *vertex form*, .
* In vertex form, you can identify the *axis of symmetry* and the *vertex* of a parabola.

|  |
| --- |
| **Vertex Form**    Axis of Symmetry is  Vertex is |

* The following is an example of *completing the square* to locate the *vertex* and the *axis of symmetry*.

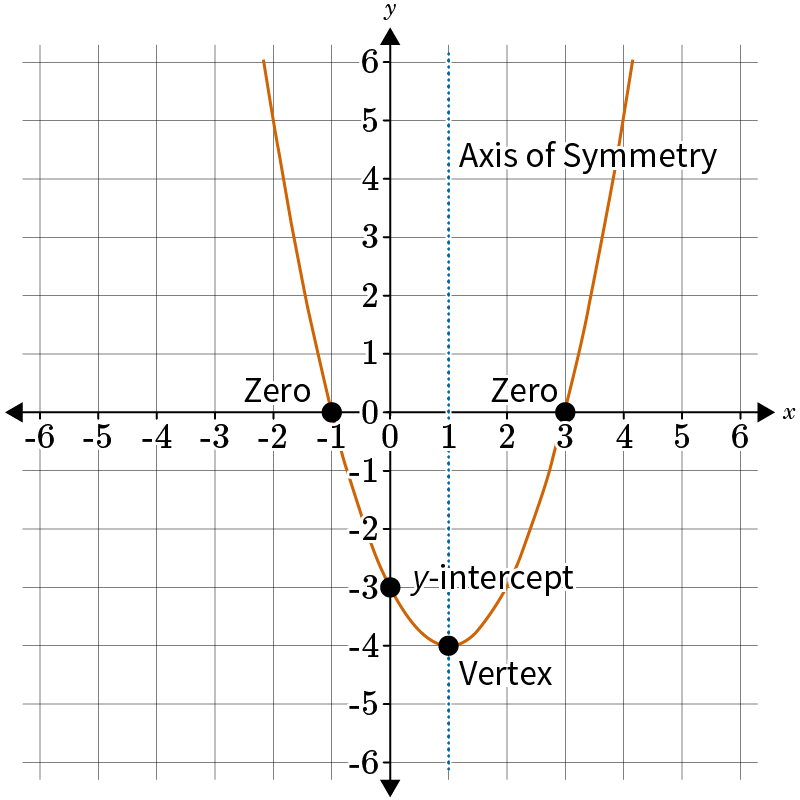
|  |  |
| --- | --- |
| **Example:** Complete the square for . Locate the vertex and axis of symmetry. | |
| **Step 1:** Move the constant term to the left side of the equation. Simplify. |  |
| **Step 2:** Divide the coefficient of by 2, and then square the result. |  |
| **Step 3:** Add the number found in step 2 to both sides of the equation. Simplify. |  |
| **Step 4:** Factor the right side now that it is a [perfect square trinomial.](#Bookmark4) | Rewrite the equation: |
| **Step 5:** Rewrite the equation to express in form. |  |
| **Step 6:** Identify the values of *h* and *k* from vertex form. | The original equation is now rewritten in vertex form:  and |
| **Step 7:** Find the vertex and axis of symmetry. | The vertex is :  The axis of symmetry is : |

**Objective 3:** In this section, you will create graphs of quadratic equations.

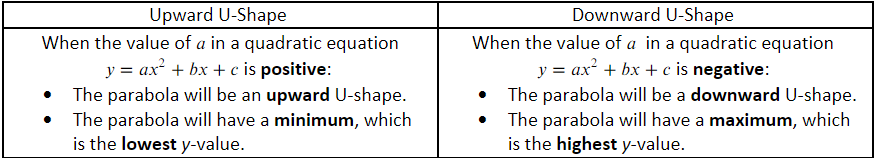
*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas:**

* [Recall](#Bookmark7) a parabola's key features and where they are on the graph.



* [Recall](#Bookmark8) the connection between a quadratic equation and the shape of the parabola.



* Use what you know about parabolas to graph a quadratic equation and identify the key features.

|  |  |
| --- | --- |
| **Example:** Create the graph of and locate the key features. | |
| **Step 1:** Interpret the shape of the parabola based on the equation. | Since the value of is –0.5, the parabola will be a downward U-shape with a maximum. |
| **Step 2:** Make a table of values for the parabola.  When generating table values:   * Find a minimum of five points * Include points that are symmetrical to ensure you capture the vertex, *x*-intercept(s), and *y*-intercept. * You can generate a table of values in GeoGebra by following these [instructions](https://www.geogebra.org/calculator/xyygsrgp). | The table of values has been generated using GeoGebra – [see the table here in table view](https://www.geogebra.org/calculator/ypc8fh4m).   |  |  |  | | --- | --- | --- | | ***x*** | ***y*** | **Key Features** | | 0 | -6 | *y*-intercept | | 1 | -2.5 |  | | 2 | 0 | *x*-intercept | | 3 | 1.5 |  | | 4 | 2 | vertex / maximum | | 5 | 1.5 |  | | 6 | 0 | *x*-intercept | | 7 | -2.5 |  | | 8 | -6 |  | |
| **Step 3:** Plot the points in the table and connect them to make a curve.  Plot *at least* five points, including the vertex, x-intercept(s), and the y-intercept. |  |
| **Step 4:** Draw the axis of symmetry and label other pieces of the graph.  [You can also graph the equation in GeoGebra and identify its key features.](https://www.geogebra.org/calculator/ypc8fh4m) |
| Notice how the corresponding points on each side of the axis of symmetry are mirror images of each other. | |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | Determine the zeros of the quadratic function . (When listing the smaller zero, remember that negative values are always smaller than positive values.)  smaller zero: 𝑥=\_\_\_\_; greater zero: 𝑥=\_\_\_ | -12; -3 |
| P 2 | Determine the zeros of the quadratic function . (When listing the smaller zero, remember that negative values are always smaller than positive values.)  smaller zero: 𝑥=\_\_\_\_; greater zero: 𝑥=\_\_\_ | 3; 13 |
| P 3 | Determine the vertex and axis of symmetry .  vertex (\_\_\_\_); axis of symmetry x = \_\_\_\_\_ | 5, -22; 5 |
| P 4 | What point is symmetrical to (−1,−3) for the graph of ? | -11, -3 |
| P 5 | What are the key points on the graph of ? Name the vertex, x-intercepts, and y-intercept.  vertex: (\_\_\_\_)  (smaller) x-intercept: (\_\_\_\_)  (larger) x-intercept: (\_\_\_)  y-intercept: (\_\_\_) | 8, -16  4, 0  12, 0  0, 48 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Use any method to locate the zeros of . | (-1, 0) (5, 0) |
| Q 2 | Complete the square to identify the vertex of *.* | (8, -50) |
| Q 3 | Identify the vertex of *.* | (-15, -18) |
| Q 4 | Create a graph; then answer the following question.  What are the key points on the graph of ? Name the vertex, x-intercept(s), and y-intercept. | *x*-intercepts: (0, 0) (8, 0)  *y*-intercept: (0, 0)  vertex: (4, -16) |
| Q 5 | What point is symmetrical to (−1,−17) for the graph of ? | (1, -17) |