Geometry

**Circles**

**Unit Summary:** This unit will guide you through the exploration of circles in the context of geometry and trigonometry. You will understand the properties of circles and their components. The unit introduces terms like radius, diameter, chord, circumference, and sector, helping you identify these components and use their relationships for problem-solving. Additionally, you'll learn about the properties of inscribed angles, central angles, quadrilaterals, arc length, and circle similarity, improving your ability to use and prove circle formulas.

**Lesson 2 – Segments Inside a Circle**

**Key Words:**

* **bisect** – to divide into two usually equal parts
* **chord** – a straight line segment joining and included between two points on a circle
* **circle** – a closed plane curve with every point on the curve equidistant from a fixed point within the curve
* **CPCTC Theorem** – the theorem that states that if two or more triangles are congruent, then their corresponding angles and sides are also congruent; it stands for "corresponding parts of congruent triangles are congruent
* **diameter** – a line segment passing through the center of a circle
* **HL Congruence Theorem** – the theorem stating that if two right triangles have congruent hypotenuses and a pair of congruent legs, then the triangles are congruent
* **perpendicular bisector** – a line or line segment that divides another line segment into two equal parts and intersects it at a 90-degree angle
* **Pythagorean Theorem** – the theorem which states that the square of the length of the hypotenuse of a right triangle is equal to the sum of the squared lengths of the other two sides
* **radius** – a line segment extending from the center of a circle or sphere to the circumference or bounding surface
* **Reflexive Property of Congruence** – the property that states that an angle, line segment, or geometric figure is congruent to itself
* **secant line** – a line that intersects a curve in two points
* **tangent line** – a line that intersects a curve at exactly one point
* **Triangle Angle Sum Theorem** – the theorem stating that the interior angles of any triangle add up to 180 degrees
* **Triangle Inequality Theorem** – the theorem stating that the sum of the lengths of any two sides of a triangle is greater than the length of the third side

**Formulas:**

* Pythagorean Theorem:
* Trigonometric Ratios: , ,

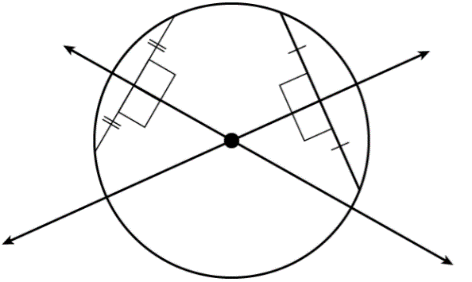
**Objective 1:** In this lesson, you will identify and use radii, diameters, and chords within circles.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas**:

* A circle is a set of all points in a plane that are *equidistant* from a center point.
* *Radii*, *diameters*, and *chords* are all lines segments related to circles.

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| Circles are named by their center point. This is circle *A*. Every point on circle *A* is the same distance from point *A* (equidistant). |  |  |
| The *radius* is a line segment with one endpoint on the circle and one endpoint at the center of the circle. |  | Key understanding: All points on a given circle are the same distance from the center point of the circle. Therefore, all radii will have the same length. |
| The *chord* is a line segment with both endpoints on the circle. |  | Key Understanding: The perpendicular bisector of any chord will pass through the center of the circle. |
| The *diameter* is a chord that passes through the center of the circle. |  | Key Understanding #1: The length of the diameter will be twice the length of the radius.  Key Understanding #2: The longest chord of a given circle is the diameter. |

* There are three related theorems about the *perpendicular bisector* of a *chord* and the center of a circle.
  + #1: If a line through the center of a circle is perpendicular to a chord, then it bisects the chord.
  + #2: If a line through the center of a circle bisects a chord, then it is perpendicular to the chord.
  + #3: The perpendicular bisector of a chord passes through the center of the circle.
* These theorems will help you find an unknown center of a circle.
  + Construct two *chords* and their *perpendicular bisectors*. The bisectors will intersect at the center of the circle.
  + 
  + Try it out in Geogebra! [Perpendicular Bisector Tool - GeoGebra](https://www.geogebra.org/geometry/v7gqhemr)
* Let’s combine the Big Ideas in the following example:

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|  | In circle C, and .  Find the length of . |
| Is a line segment from the center of the circle, making it the radius.  The radius . |
| We know that all radii in the same circle are equivalent. There is another line segment coming from the center, , another radius. |
| The diameter of a circle is always twice the length of the radius. Segment pases through the center, making it the radius.  The diameter . |
| Let’s take it a step further and find the lengths of . |
| A right triangle has been formed in the circle with sides . We know the measures of , so we can use Pythagorean Theorem to find . |
| Remember, If a line through the center of a circle is perpendicular to a chord, then it bisects the chord.  The diameter, , bisects .  Therefore, chord is split into two equal parts of .  We know that , so must be equivalent, . |

**Objective 2:** In this section, you will identify and use the relationship between the radius of a circle and a line tangent to that circle at the point where the radius intersects the circle.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

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| A line can intersect a circle at one point. This is called a *tangent line*.  Segments and rays can be tangent to circles in the same way that lines are. |  |
| A *radius* drawn from the intersection of a *tangent line* and a circle will be perpendicular to the *tangent line*. |  |
| A right angle is formed when the *radius* and *tangent line* intersect. Because of this, a right triangle can be formed.  [Recall:](#Bookmark2) When dealing with right triangles, the Pythagorean Theorem and trigonometric ratios can be used. (formulas)  [Recall:](#Bookmark1) The Triangle Angle Sum Theorem to find a missing angle in a triangle if two angles are known. (key words) |  |
| When two tangent segments are drawn to a circle from the same point, they are equal in measure. (In this example ) |  |
| Further, when radii are drawn to the tangent points, it forms two right triangles where the shown measurements are equivalent.  [Recall](#Bookmark1): Reflexive Property of Congruence, HL Congruence Theorem, and CPCTC Theorem (key words) |  |

* Tips for Solving:
  + Look for *radii, diameters,* and *chords* and make use of their definitions to solve problems.
  + Look for right triangles that are formed and utilize [formulas](#Bookmark2) to find missing angles and side lengths.
    - Pythagorean Theorem
    - Trigonometric ratios
    - Triangle Sum Theorem
  + Lable as you uncover new pieces of information.

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | *Use the image to answer the question.*    If = 13 in., then what is the length of ? | 26 in. |
| P 2 | *Use the image to answer the question.*    If = 33 cm and = 13 cm, then what is the length of to the nearest centimeter? | 30 cm |
| P 3 | *Use the image to answer the question.*    What type of line is intersecting the circle?  Option #1: a diameter  Option #2: a radius  Option #3: a secant  Option #4: a tangent | Option #3 |
| P 4 | *Use the image to answer the question.*    If has sides that are tangent to the circle, and then what is the perimeter of ? | 62 cm |
| P 5 | *Use the image to answer the question.*    If and , then what is the value of *x*? | 4 cm |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | *Use the image to answer the question.*    Use what you know about radii, chords, and diameters to identify the correct inequality. In Circle |  |
| Q 2 | *Use the image to answer the question.*    Identify a chord other than the diameter of the circle. |  |
| Q 3 | *Use the image to answer the question.*    If and , what is the length of ?   * 6 m * 16 m * 8 m * 4 m | 4 m |
| Q 4 | Use the image to answer the question.    Identify and use the relationship between the radius and a line tangent to the circle, where the radius intersects the circle, to solve the following.  If and are tangent to the circle at and , , and feet, how long is to the nearest foot?   * 51 ft. * 111 ft. * 20 ft. * 120 ft. | 111 ft. |
| Q 5 | Use the image to answer the question.    If and both equal 90 degrees, degrees, and mm, then what is the length of to the nearest millimeter?   * 7 mm * 10 mm * 20 mm * 11 mm | 7 mm |

**Lesson 3 – Angles Inside a Circle**

**Key Words:**

* **arc** – a continuous portion (as of a circle or ellipse) of a curved line or a curved path
* **central angle** – an angle with its vertex at the center of a circle
* **chord** – a straight line segment joining and included between two points on a circle
* **circumscribed angle** – an angle formed by the intersection of two tangent lines to a circle
* **diameter** – a line segment passing through the center of a circle
* **Exterior Angle Theorem** – the theorem which states that the measure of an exterior angle is equal to the sum of the measures of the two remote interior angles of a triangle
* **inscribed angle** – an angle formed by two chords that intersect on a circle
* **minor arc** – the shorter arc joining two points on a circle
* **semicircle** – a half of a circle
* **tangent line** – a line that intersects a curve at exactly one point

**Formulas:**

* Pythagorean Theorem:
* Trigonometric Ratios:

**Objective 1:** In this section, you will identify and use central angles, inscribed angles, and circumscribed angles.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas**:

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| A *central angle* on a circle is an angle with its vertex at the center of the circle. | Is the *central angle*. | |
| An *arc* is a portion of a circle created by a *central angle,* measured in degrees*.* | Minor arc: shorter arc between two points  Major arc: larger arc between two points | |
| A *central angle* and its intercepted minor *arc* have the same measure.  Key Understanding: A full circle adds up to . |  | |
| An *inscribed angle* in a circle is an angle with its vertex on the circle. | Is the *inscribed angle*. Notice that the two sides of the *inscribed angle* are *chords*. | |
| The measure of an *inscribed angle* is equal to half the measure of its intercepted *arc*. |  | |
| If two *inscribed angles* intercept the same *arc*, then they will have the same measure.  In this example, if , then . also intercepts , so . |  |  |
| A *circumscribed angle* is created when two *tangent* segments intersect outside a circle.  A *circumscribed angle* and its corresponding *central angle* are supplementary.  Recall: Supplementary angles add to . | The *central angle* is .  The *circumscribed angle* is . | |

* Tips for solving:
  + Start by labeling the given information on a diagram.
  + Look for and make use of what you know about arcs, central angles, and inscribed angles to uncover more measures.
  + Continue to label the diagram with new information as you discover it.
  + Recall that a full circle is and a half circle is .
  + It may take a few steps to find the measures of angles or arcs that are asked for in the problem.

**Objective 2:** In this section, you will understand and use the fact that an inscribed angle on a diameter measures 90 degrees.

*Mathematical Practice Standard: Look for and make use of structure.*

**Big Ideas:**

* An *arc* is a portion of a circle measured in degrees, just like angles.
* A full circle is .
* Each half of a circle is a *semicircle* that measures (half of a full circle which is ).
* *Diameters* will always divide circles in half.

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| A *central angle* on a diameter will always measure .  A *semicircle’s* central angle is the *diameter,* also measuring to .  Recall that a central angle and its intercepted arc are equal. |  |
| An *inscribed angle* on a diameter will always measure .  A *semicircle’s inscribed angle* measure is .  Recall that an inscribed angle is equal to half the measure of its intercepted angle. |  |
| An *inscribed angle* on a *diameter* forms a right triangle with the corresponding *diameter*.  [Recall:](#Bookmark3) When dealing with right triangles, the Pythagorean Theorem and trigonometric ratios can be used. (formulas) |  |

* Tips for solving:
  + Label the given information.
  + Look for arcs or angles for which you can determine the measure. Lable with this new information.
  + You may need to use many different geometric concepts when solving problems with circles.
  + It may take a few steps to find the measures of the angles, arcs, or lengths that are asked for in a problem.
  + It is always helpful to start by labeling any side lengths or angle measurements that you know.

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | *Use the image to answer the question.*    In the diagram, is  Option #1: an inscribed angle.  Option #2: a central angle.  Option #3: a circumscribed angle.  Option #4: an exterior angle. | Option #2 |
| P 2 | *Use the image to answer the question.*    If arc degrees, then what is the measure of ? | 30 degrees |
| P 3 | *Use the image to answer the question.*    If arc degrees, then what is the measure of ? | 107 degrees |
| P 4 | A circle has a diameter and an inscribed angle at vertex forming triangle . The measure of angle is 27. Using what you know about inscribed angles, what is the angle measure of ? | 63 |
| P 5 | Triangle is inscribed within a circle and has diameter line , with the inscribed angle at . If the angle measure of is 15, what is the arc measure of ? | 150 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | *Use the image to answer the question.*    In the diagram, identify .   * an inscribed angle * an exterior angle * a circumscribed angle * a central angle | an inscribed angle |
| Q 2 | *Use the image to answer the question.*    If degrees, then what is the measure of ?   * 85 degrees * 37.5 degrees * 150 degrees * 75 degrees | 75 degrees |
| Q 3 | *Use the image to answer the question.*    If degrees, then what is the measure of arc ?   * 143 degrees * 74 degrees * 53 degrees * 106 degrees | 106 degrees |
| Q 4 | Triangle is inscribed inside a circle with diameter ; the inscribed angle is . The angle measure at the vertex is 37°. Using what you understand about inscribed angles, find the arc measure of .   * 90° * 53° * 106° * 74° | 74° |
| Q 5 | Regina draws a triangle inside a circle; she labels the vertices , with the diameter as and the inscribed angle as . She draws the length of 14 cm long, and 8 cm long. Using what you know about inscribed angles, what will be the approximate angle measure of ?   * 35° * 90° * 70° * 55° | 35° |

**Lesson 4 – Circles and Triangles**

**Key Words:**

* **angle bisector** – a line or segment that divides an angle into two equal parts
* **Angle Bisector Theorem** – the theorem stating that if a point is on the bisector of an angle, then the point is equidistant from the sides of the angle
* **circle** – a closed plane curve with every point on the curve equidistant from a fixed point within the curve
* **circumcenter** – the point at which the perpendicular bisectors of a triangle’s sides intersect; it also is the center of a circumscribed circle of that triangle
* **circumscribe** – to draw a shape around a figure so that the shape intersects the vertices of the figure
* **circumscribed circle** – a circle constructed around a figure so as to touch as many points as possible
* **construct** – to create a shape or an object in geometry using appropriate tools
* **equidistant** – when two given points are the same or equal distance from a third point
* **incenter** – the single point in which the three bisectors of the interior angles of a triangle intersect and which is the center of the inscribed circle
* **inscribe** – to draw within a figure so that the drawing inside the figure touches the figure in as many places as possible
* **perpendicular bisector** – a line or line segment that divides another line segment into two equal parts and intersects it at a 90-degree angle
* **tangent line** – a line that intersects a curve in exactly one point
* **triangle** – a closed plane figure bounded by straight lines

**Objective 1:** In this section, you will construct the inscribed circle of a triangle.

*Mathematical Practice Standard: Use appropriate tools strategically.*

**Big Ideas**:

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| To *inscribe* a circle in a triangle means to find the largest circle that can be contained within the triangle. |  |
| The center of the *inscribed circle* is called the *incenter* of the triangle.  Recalling what you know about the radius and the center of a circle, the *incenter* is *equidistant* from all the sides of the triangle. |  |
| Each of the three sides of the triangle are *tangent* lines. They each intersect the triangle at exactly one point.  Each side of the triangle is perpendicular to the radius of its *inscribed circle*. |  |
| The triangle's *incenter* is at the intersection of the three *angle bisectors*.  [Recall](#Bookmark4): Angle Bisector Theorem |  |

* GeoGebra tools:
  + [Incircle – GeoGebra](https://www.geogebra.org/m/fkh76DQ4)
  + [Construction: Circumscribed and Inscribed Circles – GeoGebra](https://www.geogebra.org/m/c2wpvfbp)

**Objective 2:** In this section, you will construct the circumscribed circle of a triangle.

*Mathematical Practice Standard: Use appropriate tools strategically.*

**Big Ideas:**

* In a *circumscribed circle* of a triangle, the circle is formed around the triangle and passes through each of the triangles' three vertices, or points.

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| In a *circumscribed circle* of a triangle, the circle is formed around the triangle and passes through each of the triangle’s vertices. |  |
| The center of a *circumscribed circle* is called the *circumcenter*. |  |
| The *circumcenter* is the intersection of the [*perpendicular bisector*](#Bookmark4)of each side of the triangle. |  |
| The radius of the circle is the distance between the *circumcenter* and any of the triangle’s vertices.  Using the radius, the circle is formed by connecting all three vertices around the circumcenter. |  |

* Follow these steps to construct a circumscribed circle of a triangle:
  + Step 1: Find the *perpendicular bisector* of each side of the triangle.
  + Step 2: Draw a point where all three bisectors intersect, this is the *circumcenter*.
  + Step 3: Measure the distance between the circumcenter and one of the vertices of the triangle.
  + Step 4: Use a compass or online graphing tool to construct the circle being sure it passes through all three vertices.
* Geogebra tools:
  + [Circumscribed Circle – GeoGebra](https://www.geogebra.org/m/h2czjucd#material/uctrekw6)
  + [Construction: Circumscribed and Inscribed Circles – GeoGebra](https://www.geogebra.org/m/c2wpvfbp)

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | At what intersection point is the incenter of a triangle located?  Option #1: the three angle bisectors  Option #2: the three perpendicular bisectors  Option #3: the three medians  Option #4: the three altitudes | 1 |
| P 2 | When a circle is inscribed in a triangle, which part of an inscribed circle is each side of the triangle perpendicular to?  Option #1: a chord  Option #2: an arc  Option #3: a radius  Option #4: a secant | 3 |
| P 3 | *Use the image to answer the question.*    What type of rays are shown in the image?  Option #1: angle bisectors  Option #2: perpendicular bisectors  Option #3: altitudes  Option #4: medians | 1 |
| P 4 | What is the first step to construct the circumscribed circle of a triangle?  Option #1: Create perpendicular bisectors of the sides of the triangle.  Option #2: Create midsegments to connect the sides of the triangle.  Option #3: Use a compass to draw a circle around the triangle.  Option #4: Use the protractor to create angle bisectors of the triangle. | 1 |
| P 5 | A figure’s vertices are \_\_\_\_\_\_\_\_\_\_\_\_ from the circumcenter.  Option #1: the absolute value  Option #2: equidistant  Option #3: halfway  Option #4: opposite reciprocals | 2 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | If you construct the inscribed circle of a triangle, then what is the name for the point where the three angle bisectors of a triangle intersect?   * the circumcenter * the centroid * the orthocenter * the incenter | the incenter |
| Q 2 | What part of a triangle can the incenter be equidistant to?   * a side * a vertex * a midpoint * a side or vertex | a side |
| Q 3 | If you construct a circumscribed circle around a triangle, what does the intersection of the perpendicular bisectors create?   * an incenter * a centroid * an orthocenter * a circumcenter | a circumcenter |
| Q 4 | Which of the following diagrams shows a circle circumscribed about a triangle? |  |
| Q 5 | In a circle circumscribed about a triangle, the distance from any vertex to the circumcenter is   * congruent. * greater than one. * less than one. * similar | congruent |

**Lesson 5 – Circles and Quadrilaterals**

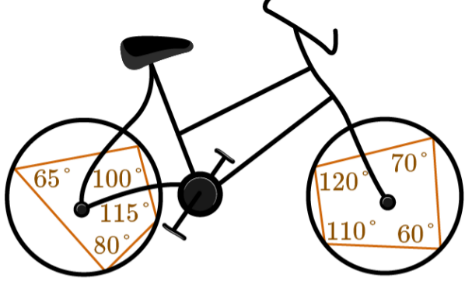
**Key Words:**

* **cyclic polygon** – a polygon whose vertices all lie on a circle
* **inscribed angle** – an angle formed by two chords that intersect on a circle
* **Inscribed Quadrilateral Theorem** – a theorem that states that a quadrilateral can be inscribed in a circle if and only if the opposite angles of the quadrilateral are supplementary
* **quadrilateral** – a polygon of four sides
* **supplementary** – a pair of angles whose sum is equal to 180 degrees

**Objective 1:** In this section, you will prove and apply properties of opposite angles within a quadrilateral inscribed in a circle.

*Mathematical Practice Standard: Look for and express regularity in repeated reasoning.*

**Big Ideas**:

* Polygons other than triangles can be *inscribed* within circles such as pentagons and hexagons.
* Polygons *inscribed* in a circle have special angle properties.
  + *Inscribed Quadrilateral Theorem*: opposite angles of a quadrilateral in an inscribed circle are *supplementary*.
  + Recall that *supplementary* means two angles that add to .
  + For example, in this image, you will see that the opposite angles add up to .
    - 
* Sometimes, you will only be given the measure of the circles arc when solving these types of problems.
  + Recall that a full circle is and a semicircle or half-circle is .
  + Recall the relationship between *inscribed angles* and *intercepted arcs*.

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|  | Angle B is half the measure of arc AC.  Arc AC is twice the size of the inscribed angle B. |  |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Use the image to answer the question.    Apply properties of quadrilaterals inscribed in a circle to determine the if and , while the .  \_\_\_ | 48 |
| P 2 | Quadrilateral is inscribed in circle . The and . Determine the . | 21 |
| P 3 | A quadrilateral is inscribed in a circle. Two consecutive angles measure 88° and 108°. The angle opposite the 108° angle is labeled . Find the value of *x*.  *x* = \_\_\_\_° | 72 |
| P 4 | A quadrilateral is inscribed in a circle. Two consecutive angles measure 88° and 108°. The angle opposite the 88° angle is labeled *y*. Find the value of *y*.  *y* = \_\_\_\_° | 92 |
| P 5 | *Use the image to answer the question.*    The image shows an inscribed quadrilateral. The . Determine the . | 128 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Opposite angles of an inscribed quadrilateral can be proven to be   * acute. * congruent. * supplementary. * complementary. | supplementary |
| Q 2 | Quadrilateral is inscribed in circle . Given and , find and .   * and * and * and * and | and |
| Q 3 | Identify the true statement about inscribed angles and their intercepted arc.   * The intercepted arc is half the measure of the inscribed angle. * The intercepted arc is twice the measure of the inscribed angle. * The inscribed angle is twice the measure of its intercepted arc. * The intercepted arc is equal in measure to its inscribed angle. | The intercepted arc is twice the measure of the inscribed angle. |
| Q 4 | Draw the following diagram: quadrilateral inscribed in circle . The , , and . Select the true statement. |  |
| Q 5 | Quadrilateral is inscribed in circle . Opposite angles and measure and , respectively. Determine the measure of the smaller angle.   * 24 * 20 * 56 * 40 | 40 |

**Lesson 6 – Similar Circles**

**Key Words:**

* **circumference** – the distance around a circle
* **dilation** – the act or process of expanding or reducing a shape by the same scale factor
* **radius** – a line from the center to the circumference of a circle
* **scale factor** – a number representing the ratio between the dimension of an object and the dimension of a dilation of that object
* **similar figures** – figures with congruent corresponding angles and proportionate corresponding sides

**Formulas**:

* Scale factor of a dilated circle:

**Objective 1:** In this section, you will prove that all circles are similar.

*Mathematical Practice Standard: Look for and express regularity in repeated reasoning.*

**Big Ideas**:

* All circles are *similar figures* to each other because a circle is a circle, not matter the size.
* When a figure, such as a circle, undergoes a *dilation,* it creates a similar figure (bigger or smaller).
* The ratio used to dilate a figure is called a *scale factor*.

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| In circles, the *scale factor, a,* is defined as the ratio of the new *radius* to the original *radius.*  If the scale factor is greater than 1, the dilation produces a larger circle.  If the scale factor is less than 1, the dilation produces a smaller circle.  If the scale factor is equal to 1, the circles are congruent (same size). | Using the circumference:  , where ,  and |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | A circle with a radius of 6 inches is dilated by a scale factor of 4. What is the radius of the transformed circle? | 24 in. |
| P 2 | If circle *O* has a circumference of 37 meters and circle *P* has a circumference of 111 meters, then what is the scale factor from *O* to *P*? | 3 m |
| P 3 | If circle *A* has a radius of 6 millimeters, with a circumference of approximately 38 millimeters, and circle *B* has a radius of 60 millimeters, then what is circle *B*’s approximate circumference. | 380 mm |
| P 4 | If circle *X* is constructed so that it has a circumference of 120 centimeters, and is dilated to create circle with a circumference of 90 centimeters, then what is the scale factor? Enter any non-whole number as a fraction. | cm |
| P 5 | If the radius of circle is 13 feet, and the radius of circle is 78 feet, then what is the scale factor from circle to circle ? Enter any non-whole number as a fraction. | 6 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | You can prove that two circles are similar to each other because the ratio of radius to radius taken anywhere within two circles is uniform. What is the process of growing or shrinking a circle from one to the other called?   * dilation * translation * rotation * reflection | dilation |
| Q 2 | Dilations are transformations that produce similar figures; therefore, all circles are   * congruent. * parallel. * similar. * concentric. | similar |
| Q 3 | Amias is trying to prove that any two given circles are similar. He starts by drawing circle with a radius of *a* and circle with a radius of *b* where . He then translates circle so that circle and circle have the same center. How can he complete his proof to show that circle is similar to circle ?   * Amias can dilate circle by a factor of and show that the circles now coincide. Because a sequence of transformations maps circle onto circle , circle is similar to circle . * Amias can dilate circle by a factor of and show that the circles now coincide. Because a sequence of transformations maps circle onto circle , circle is similar to circle . * Amias can rotate both circle A and circle B around their center points until they coincide. Because a sequence of transformations maps circle onto circle , circle is similar to circle . * Amias can rotate circle around its center point until it coincides with circle . Because a sequence of transformations maps circle onto circle , circle is similar to circle . | Amias can dilate circle by a factor of and show that the circles now coincide. Because a sequence of transformations maps circle onto circle , circle is similar to circle . |
| Q 4 | If the circumference of circle is and the circumference of is , then what is the scale factor from to ? |  |
| Q 5 | If the radius of circle is 14 meters and it is dilated by a scale factor of 2.5, then what will be the circumference of circle ?   * 220 m * 10 m * 345 m * 44 m | 220 m |

**Lesson 7 – Trigonometry and Circles**

**Key Words:**

* **area** – the amount of space taken up by a two-dimensional shape
* **circumference** – the distance around a circle
* **diameter** – a line segment passing through the center of a circle
* **perimeter** – the continuous line forming the boundary of an enclosed geometric figure
* **radius** – a line from the center to the circumference of a circle
* **ratio** – a mathematical comparison of two quantities
* **regular polygon** – a polygon that is both equilateral and equiangular
* **scale factor** – a number representing the ratio between the dimension of an object and the dimension of a dilation of that object
* **similar** – proportional in shape but not necessarily the same size

**Formulas**:

* The circumference of a circle:
* The radius of a circle, given the circumference:
* The area of a circle:
* The radius of a circle, given the area:

**Objective 1:** In this section, you will describe an informal argument for the formula for the circumference of a circle.

*Mathematical Practice Standard: Look for and express regularity in repeated reasoning.*

**Big Ideas**:

|  |
| --- |
| The *ratio* of the *circumference* of a circle to its *diameter* is represented by the irrational number , which can be approximated to 3.14.  The equation for the *circumference* of a circle: |
| When given the *circumference* of a circle, you can rearrange the formula for *circumference* to solve for *r*. |
| Recall that the radius of a circle is half of the diameter.  , and thus |

**Objective 2:** In this section, you will describe an informal argument for the formula for the area of a circle.

*Mathematical Practice Standard: Look for and express regularity in repeated reasoning.*

**Big Ideas:**

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| --- |
| The equation for the *area* of a circle is: |
| To find the *area* given the *radius*, you can rearrange the *area* equation for *r*: |
| Recall that the radius of a circle is half of the diameter.  , and thus |

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | If the radius of a circle is 7 cm, then what is the circumference? Round to the nearest whole number. | 44 cm |
| P 2 | If the circumference of a circle is , then how long is the diameter? | 20 |
| P 3 | If a radius is inch, then what is the circumference of the circle rounded to the nearest tenths place?  The circumference is \_\_\_\_ inches. | 1.6 |
| P 4 | If a circular table is 6 feet across the middle, approximately how much fabric would be needed to cover the top of the table? Round to the nearest square foot.  \_\_\_ ft.2 | 28 |
| P 5 | If the area of a circle is , then what is the length of the diameter? Round to the nearest whole number.  \_\_\_\_\_ cm | 22 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | Which term would describe the distance around a circle?   * the radius * the circumference * the diameter * the chord | the circumference |
| Q 2 | If , then what is the circumference of a circle when the radius is 97 m? Round to the nearest whole number.   * 194 m * 609 m * 305 m * 9,409 m | 609 m |
| Q 3 | If the circumference of a circle is 27 mm, then what is the diameter to the nearest tenth of a mm?   * 8.6 mm * 4.3 mm * 17.2 mm * 84.8 mm | 8.6 mm |
| Q 4 | Describe the formula for finding the area of a circle: multiply π by the length of   * an inscribed angle. * a radius squared. * a diameter. * a tangent. | a radius squared. |
| Q 5 | If the circumference of a circle is 198 feet, then what is the area to the nearest square foot?   * 99 ft.2 * 63 ft.2 * 32 ft.2 * 3, 120 ft.2 | 3, 120 ft.2 |

**Lesson 8 – Radian Angle Measure**

**Key Words:**

* **arc** – a continuous portion (as of a circle or ellipse) of a curved line or a curved path
* **circumference** – the distance around a circle
* **constant of proportionality** – the ratio between two quantities that are directly proportional
* **inscribed angle** – an angle formed by two chords that intersect on a circle
* **proportion** – the equating of two ratios
* **proportional** – having the same or a constant ratio
* **radian** – a unit of plane angular measurement that is equal to the angle at the center of a circle subtended by an arc whose length equals the radius, or approximately 57.3 degrees
* **radius** – a line from the center to the circumference of a circle

**Formulas:**

* Arc length:
* Radius given the arc length and central angle:
* Central angle given the arc length and radius:
* Angle measure in radians:
* Arc length given the angle and the radius:
* Radius given the angle and the arc length:

**Objective 1:** In this section, you will derive the fact that the length of the arc of a circle intercepted by an angle is proportional to the radius of the circle.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas**:

* Recall that an *arc* of a circle is formed by an *inscribed angle*, .Also that a central angle is formed by two radii meeting at the center of the circle.

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| The inscribed angle, , makes an arc of length around the outside of the circle.  The *arc* length of a whole circle is its *circumference*.  The *arc length* is proportional to the *radius* of a circle.  You can calculate the *arc* length given the radius and *inscribed angle* measure. |  |
| You can also find the *radius* if you are given the *arc length*.  Rearrange the equation for the *arc length* to solve for *r*. |  |
| You can also find the *central angle*, , given the *arc length* and the *radius*.  Rearrange the equation for *arc length* to solve for . |  |

**Objective 2:** In this section, you will define the measure of an angle in radians as the ratio of the arc length created by the angle to the circle’s radius.

*Mathematical Practice Standard: Reason abstractly and quantitatively.*

**Big Ideas:**

* *Radians* are another way of measuring angles, other than degrees.
* *Radians* are used to measure *arc lengths* because they give you more exact values than degrees.
* The measure of an angle that represents the entire circle, , is .

|  |  |  |
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| *Radians* are defined by the proportional relationship between an angle’s *arc length* and the *radius* of the circle.  A measure of one *radian* is always formed when the *radius* is equal to the arc length intersecting that angle.  \*One radian = |  |  |
| The relationship between the *arc length*, *radius*, and *angle* measure in *radians* can be written as:  Where *s* is the arc length, *r* is the radius, and is the *angle* measure.  ***Remember, when using , there is no need to find a decimal answer. Keeping in your answer ensures that it is as exact as possible.*** | | |
| Depending on what you need to solve for, you can use different variations of the equation.  To find the *arc length* given the *angle* and the *radius*:  To find the *radius* given the *angle* and the *arc length*: | | |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | If the central angle of a circle measures 42 degrees and the radius is 14 inches long, then how long is the arc intercepted to the nearest inch?  \_\_\_\_ in. | 10 |
| P 2 | A circle with a circumference of 125 meters has a central angle of 72 degrees. What is the length of the arc it creates?  \_\_\_\_ m | 25 |
| P 3 | If a circle has a circumference of and a central angle of 90 degrees is drawn, then what is the length of the minor arc? |  |
| P 4 | If a circle has a radius of 6 cm and a central angle of , then what is the length of the arc created by the angle? |  |
| P 5 | If a pizza with a 24-inch diameter is cut into 12 equal pieces, then what is the length of the crust along the edges that are cut? |  |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | To derive the fact that the length of the arc of a circle intercepted by an angle is proportional to the radius of the circle, what should the arc length first be compared to?   * the circumference * the diameter * the area * a chord | the circumference |
| Q 2 | If the arc of a circle is 15 meters long and the radius of the circle is 3 meters, then what is the measure of the central angle, to the nearest tenth of a degree, that intercepts that 15-meter arc?   * 353.4 degrees * 7.9 degrees * 282.7 degrees * 286.5 degrees | 286.5 degrees |
| Q 3 | The measure of an angle in radians is the ratio of the arc length created by the angle to the circle’s radius. If the radius is 6 feet and the arc length is 2π, then define the angle measure in radians. |  |
| Q 4 | If the radius of a circle is 10 feet and the central angle is , then what is the arc length in radians? |  |
| Q 5 | If a central angle of is created with two radii that are 30 inches long, then how long is the arc they will cut in radians? |  |

**Lesson 9 – Area of a Sector**

**Key Words:**

* **arc** – a continuous portion (as of a circle or ellipse) of a curved line or a curved path
* **area** – the amount of space taken up by a two-dimensional space
* **radius** – a line from the center to the circumference of a circle
* **sector** – a space inside a circle created by an arc and two radii

**Formulas:**

* Area of a circle:
* Area of a sector:
* Radius given the diameter:

**Objective 1:** In this section, you will derive the formula for calculating the area of a sector.

*Mathematical Practice Standard: Look for and express regularity in repeated reasoning.*

**Big Ideas**:

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| Recall that a straight line will always measure .  To make a full circle, you must draw a full arc across both sides of a straight line.  This creates a full . Therefore, every circle has an angle measurement of . |  |
| Recall the formula for the area of a circle.  This formula is used to find the area inside of a full circle. |  |
| You can apply the area formula to find a smaller portion of the *area*, known as the *sector*.  A *sector* is a portion of a circle’s full angle of .  To find the *area* of a *sector*, you need the *radius* and the angle formed by the *sector*. | Remember, it is more precise to leave the solution in terms of *pi (*), unless stated otherwise. |

* Geogebra tool:
  + [Area of a sector. – GeoGebra](https://www.geogebra.org/m/FSqNDNxN)

**Objective 2:** In this section, you will use a formula to calculate the area of a sector on a circle.

*Mathematical Practice Standard: Attend to precision.*

**Big Ideas:**

|  |  |
| --- | --- |
| Recall that the *area* of a *sector* is the space inside a circle created by an *arc* and two *radii*. |  |
| Recall the formula for the area of a sector: | You need two pieces of information to find the sector area:  = the measure of the angle created  = radius  Recall that the *radius* is always the same length no matter where it is on the circle. |
| Sometimes, you will be given the *diameter* instead of the *radius*.  Recall that the *radius* is half of the *diameter*.  See this example for a reference -> | Imagine you buy a pizza with a 12-inch diameter and eat two out of six slices. The missing slices of pizza form an angle measuring . What is the area of the pizza you ate?    We can apply the formula for the area of a sector given that we have the angle measure () and the radius. The problem gives us the diameter, so we just need to do the math to find the radius.  Now, we can apply the formula for the area of a sector to solve. |

* Geogebra tool:
  + [Area of a sector. – GeoGebra](https://www.geogebra.org/m/FSqNDNxN)

**Practice Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| P 1 | What is the area of the sector of a circle with and a diameter of 18 inches? Write your answer to two decimal places.  \_\_\_\_\_ | 27.45 |
| P 2 | If a circle has a diameter of 20 inches, what is its radius?  \_\_\_\_\_ inches | 10 |
| P 3 | Danae was asked to find the area of a sector with and a radius of 23 meters. She calculated 40.14 m2 for the area of the sector. What was her mistake?  Statement #1: She used the diameter instead of the radius.  Statement #2: She incorrectly calculated .  Statement #3: She forgot to square the radius.  Statement #\_\_\_\_\_ describes Danae’s mistake. | 3 |
| P 4 | A circle has a radius of 6 ft. Find the area of a sector if . Leave your answer in terms of .  in.2 | 15 |
| P 5 | Kelsey is given a circle with a radius of 5 cm. She calculates the area of a sector that has an angle of 216 and provides the solution of cm2. What was her mistake?  Statement #1: She incorrectly calculated .  Statement #2: She forgot to square the radius.  Statement #3: She calculated the circumference instead of the area.  Option #\_\_\_\_\_ describes Kelsey’s mistake. | 2 |

**Quick Check Questions and Answers**

|  |  |  |
| --- | --- | --- |
|  | Question | Answer |
| Q 1 | How do you derive the formula for the area of a sector?   * Divide the measurement of the sector’s angle by , then divide the quotient by π times the radius squared. * Divide the measurement of the sector’s angle by , then multiply the quotient by π times the radius. * Divide the measurement of the sector’s angle by 18, then multiply the quotient by π times the radius squared. * Divide the measurement of the sector’s angle by , then multiply the quotient by π times the radius squared. | Divide the measurement of the sector’s angle by , then multiply the quotient by π times the radius squared. |
| Q 2 | What is the area of the sector of a circle with and a radius of 4 feet? Include pi in your calculation.   * ft.2 * ft.2 * ft.2 * ft.2 | ft.2 |
| Q 3 | Using the formula for the area of a sector, solve for the area of a sector with an angle of 135° and a radius of 13 in.   * in.2 * in.2 * in.2 * in.2 | in.2 |
| Q 4 | The radius of a circle is 6 in. Find the area of a sector with an angle of 120°.   * in.2 * in.2. * in.2 * in.2 | in.2 |
| Q 5 | There is a sprinkler in Amut’s backyard that can shoot water a distance of 15 feet from the sprinkler head. It rotates on the center point at an angle of 180° before returning to its starting position. What area of the backyard, in feet, can the sprinkler spray?   * ft.2 * ft.2 * ft.2 * ft.2 | ft.2 |