Math Course

**Modeling with Geometry**

**Unit Summary:** In this unit, you will learn how to model with geometry in calculating the area and density of 2D and 3D objects. You will calculate the area of an object that can be modeled by a single shape or multiple copies of a single shape. You will also find the area of objects that are modeled by composite shapes. Finally, you will calculate the surface area of an object that can be modeled by a prism, cylinder, pyramid, or a cone.

**Lesson 2 – Area of a Model**

**Key Words:**

* **area** – the number of square units needed to cover a two-dimensional figure
* **composite figure** – a two-dimensional figure made up of multiple two-dimensional figures

**Formulas:**

* Area of a Triangle: $A=\frac{1}{2}bh$
* Area of a Square/Rectangle/Parallelogram: $A=bh $
* Area of a Circle: $A=πr^{2}$
* Area of a Trapezoid: $A=\frac{1}{2}\left(b\_{1}+b\_{2}\right)h$
* Trigonometric Ratios:
	+ $\sin(θ)=\frac{opp}{hyp}$
	+ $\cos(θ)=\frac{adj}{hyp}$
	+ $\tan(θ)=\frac{opp}{adj}$

**Objective 1:** In this section, you will calculate the area of objects that can be modeled by a shape, or that can be modeled by multiple copies of that shape.

*Mathematical Practice Standard: Model with mathematics.*

**Objective 2:** In this section, you will calculate the area of objects that can be modeled by composite figures.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* Each geometric shape has its own set of formulas to find *area*, perimeter, and/or volume.
* The calculations you perform when finding an object's *area* depend on the shapes chosen to model the object.
	+ Recall the [formulas](#Bookmark1) for areas of certain shapes.
	+ Recall [trigonometric ratios](#Bookmark1) when working with triangles.
* Some models are simple, like finding the *area* of a bicycle tire or a window.
* Some models are much more complex and require combining the *area* of several different shapes. These models are called *composite figures*.
	+ These shapes include squares, rectangles, triangles, circles, etc.
* When you encounter a shape that does not have an *area* formula, a *composite figure*, there are two methods of approach.

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| Method 1: Break It downThis method involves dividing the composite figure into smaller shapes with areas that can be easily calculated. |
| Problem: Find the area of this shape. |
| Step 1: Break the shape into smaller shapes. | This shape can be divided into a square (in the middle) and four triangles (around the outside). |
| Step 2: Calculate the area of the smaller shapes. | Using the grid, we can determine the measurements of the square and the triangles to be able to calculate the area.All **four** triangles are congruent, with a base of 2 inches and a height of 3 inches. The square has a base and height of 2 inches.  |
| Step 3: Sum the areas of all of the smaller shapes. | You can find the area of the entire star by adding the area of the square and the area of the **four** triangles. |

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| Method 2: Build It UpThis method involves enclosing a composite figure inside of a larger shape and subtracting the areas of the extra pieces.  |
| Problem: Calculate the area of the figure. |
| Step 1: Enclose the figure inside of a large rectangle. |  |
| Step 2: Find the area of the large rectangle. |  |
| Step 3: Bread up the pieces that are not part of the shape. | The pieces that are not part of the original shape are three triangles and one rectangle. |
| Step 4: Calculate the area of each piece. |  |
| Step 5: Subtract the areas of the triangles and the small rectangle from the area of the large rectangle found in Step 2. |  |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Michael has 34 square tiles left over from a previous project. Each tile has a side length of 6 inches. If Eric wants to tile his bathroom floor, what is the maximum number of square inches that he can cover with the amount of tile he has?\_\_\_\_\_ in.2 | 1,224 |
| P 2 | Shashi is painting a decorative circle on her patio. She wants the outer circle to have a diameter of 10 ft. and the inner circle to have a diameter of 7 ft. If she paints only the area between these circles, what will be the area of the painted section? Include the full value of pi in your calculation and round your answer to the nearest hundredth. \_\_\_\_\_ft.2 | 40.06 |
| P 3 | *Use the image to answer the question.*A rectangle measures 4 inches in height and 10 inches in length. A semicircle is drawn at the right end of a rectangle, sharing a 4-inch side. A point is drawn at the center of the shared side.Find the area of the figure. Use $π=3.14$ and round your answer to the nearest hundredth if required.\_\_\_\_\_ in.2 | 46.28 |
| P 4 | *Use the image to answer the question.*Two rectangles are drawn side by side, sharing a common side. Find the area of the figure. Round your answer the nearest hundredth if required.\_\_\_\_\_ in.2 | 28 |
| P 5 | *Use the image to answer the question.*A double-ended arrow.Find the area of the figure. Round your answer to the nearest hundredth if required.\_\_\_\_\_ in.2 | 64 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | Constantinos is painting a rectangular outline of a frame on the wall, leaving the area inside unpainted. He wants the outside of the frame to be 9 ft. wide by 12 ft. tall. The inside of the frame will be 6 ft. wide by 10 ft. tall. Calculate the area of the frame. | 48 ft.2 |
| Q 2 | Shavonne is decorating a wreath that has an outer diameter of 22 in. and an inner diameter of 18 in. What is the maximum area of the wreath that she can cover with decorations? | 125.66 in.2 |
| Q 3 | *Use the image to answer the question.*A rectangle and a parallelogram are drawn side by side on a grid and share a common side. Which of the following correctly calculates the area of the model? | 120 ft.2 |
| Q 4 | *Use the image to answer the question.*A trapezoid, a square, and a semicircle are drawn connected. Which of the following correctly calculates the area of the figure, rounded to the nearest hundredth? Use $π=3.14.$ | 19.53 in.2 |
| Q 5 | *Use the image to answer the question.*A pentagon is divided into three right triangles and a trapezoid.Which of the following correctly calculates the area of the figure? | 39.5 in.2 |

**Lesson 3 – Surface Area of a Model**

**Key Words:**

* **apex** – the point on a cone or a pyramid that is furthest from the base; the point where the triangular faces of a pyramid meet
* **cone** – a solid with one circular base and a curved lateral surface that comes to an apex point
* **cylinder** – a solid figure with two congruent round, flat bases and a curved surface connecting them
* **lateral surface area** – the combined area of the side surfaces of a solid
* **prism** – a solid with two parallel polygonal bases connected by parallelogram-shaped lateral faces
* **pyramid** – a solid with one base and triangle-shaped lateral faces which join at the apex
* **regular polygon** – a polygon that is both equilateral and equiangular
* **regular pyramid** – a pyramid whose base is a regular polygon
* **right cone** – a cone whose apex is directly above the center of its base
* **right cylinder** – a cylinder whose lateral face and bases are perpendicular
* **right prism** – a prism whose lateral faces and bases are perpendicular
* **right pyramid** – a pyramid whose apex is directly above the center of its base
* **slant height** – the altitude of a side of a regular right pyramid; the distance from the apex of a cone to the outer rim of its base
* **surface area** – the total area of the surface of a solid

**Formulas:**





* Pythagorean Theorem: $a^{2}+b^{2}=c^{2}$

**Objective 1:** In this section, you will calculate the surface area of objects that can be modeled by prisms or cylinders.

**Objective 2:** In this section, you will calculate the surface area of objects that can be modeled by pyramids or cones.

*Mathematical Practice Standard: Model with mathematics.*

**Big Ideas**:

* The *total surface area* of a solid is the sum of the areas of all its faces, including the bases.
* The *lateral surface area* of a solid is the area of just its side faces, NOT the bases.
* Recall that the perimeter is the total distance around a 2D shape that makes up the base of the solid. In other words, add the lengths of the sides of the base together.
* Recall the [formulas](#Bookmark1) for areas of 2D shapes. The formula you use for the area will depend on the shape of the base of the solid.
* Height is measured perpendicular to the figure's base, while *slant height* is measured diagonally along the figure's surface.
* You may need to use the [Pythagorean Theorem](#Bookmark2) if the slant height is not given.

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| Cylinder | $$SA\_{lateral}=2πrh$$Where $r $is the radius and $h $is the height. | $$SA\_{total}=2πrh+2πr^{2}$$Where $r $is the radius and $h $is the height. |
| Prism | $$SA\_{lateral}=P\_{base}⋅h$$Where is the $P $ perimeter and $h $ is the height. | $$SA\_{total}=P\_{base}⋅h+2⋅A\_{base}$$Where is the $P $ perimeter,$h $ is the height, and $A $ is the area. |
| Pyramid | $$SA\_{lateral}=\frac{1}{2}\left(P\_{base}\right)l$$Where $P $is the perimeter and $l $is the slant height. | $$SA\_{total}=\frac{1}{2}\left(P\_{base}\right)l+A\_{base}$$Where $P $is the perimeter, $l $is the slant height, and $A $is the area.  |
| Cone | $$SA\_{lateral}=πrl$$Where is the $r $radius and is the $l $slant height. | $$SA\_{total}=πrl+πr^{2}$$Where $r $is the radius and $l $is the slant height. |

* Tips for calculating surface area:
	+ Sometimes, a cylinder or prism in the real-world might be oriented differently. The “base” and the “height” may not appear differently if a solid is laying on its side. Always think carefully about what dimensions you need for the calculations you are doing.
	+ Always write out the formula you need to use depending on the object's shape. Then, work on identifying the measurements needed to complete the problem.
	+ Similar to the area, the surface area is measured in square units ($cm^{2}, in^{2}, ft^{2}$, etc.)

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | Determine the surface area of a prism with a triangular base. The base is an isosceles triangle with a base length of 12 in. and a height of 8 in. The height of the prism is 14 in.\_\_\_\_\_ in.2 | 544 |
| P 2 | Find the lateral surface area of a square base prism with a length of 6 ft. and a height of 12 ft.\_\_\_\_\_ ft.2 | 288 |
| P 3 | Find the surface area of a cylinder with a base radius length of 5 ft. and a height of 8 ft. Use π = 3.14.\_\_\_\_\_ ft.2 | 408.2 |
| P 4 | Determine the surface area of a pyramid with a square base. The side of the square base is 3 in. and the slant height of the pyramid is 5 in.\_\_\_\_\_ in.2 | 39 |
| P 5 | Determine the surface area of a cone if the radius of the base is 8 in. and the slant height of the cone is 10 in. Use π = 3.14 and round your answer to the nearest hundredth.\_\_\_\_\_ in.2 | 452.16 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | A base of a prism is a trapezoid that has one base length of 4 in., another base length of 10 in., and a height of 4 in. The height of the prism is 10 in. Which of the following correctly calculates the surface area of the prism with an isosceles trapezoid base? | 296 in.2 |
| Q 2 | Which of the following correctly calculates the lateral area of a cylinder with a base radius length of 7 in. and a height of 10 in? Use π = 3.14. | 439.6 in.2 |
| Q 3 | Peter plays renaissance drums, which are cylindrical in shape, in military reenactments in his hometown. He just bought a new drum with the money he has saved from working. Peter’s new drum has a diameter of 13 inches and a height of 19 inches. What is the approximate surface area of Peter’s drum? | 1,041.44 in.2 |
| Q 4 | Which of the following formulas correctly calculates the surface area of a cone? | $$πrl+πr^{2}$$ |
| Q 5 | A rectangular pyramid has a base with sides 10 ft. and 7 ft. The slant height where the base has the longer side is 7 ft., and the slant height where the base has the shorter side is 8 ft. Which of the following correctly calculates the surface area of the pyramid? | 196 ft.2 |

**Lesson 4 – Density in Two Dimensions**

**Key Words:**

* **density –** a value that describes how compact or concentrated something is; the average number of units per unit of space

**Formulas:**

* 2D Density Formula: $d=\frac{total \# of objects}{total area}$

**Objective 1:** In this section, you will calculate the density of two-dimensional objects.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Objective 2:** In this section, you will use the density calculations of two-dimensional objects to solve problems.

*Mathematical Practice Standard: Attend to precision.*

**Big Ideas**:

* Density is a value that describes how much of something is in a given unit of space. This unit will focus on density in a two-dimensional space.
* Common example of two-dimensional density:
	+ Population density - measures how crowded an area is
	+ Planting density – measures how many plants can be planted in an area
* It can be modeled with the formula: $d=\frac{total \# of objects}{total area}$.
	+ The solution is stated in the form of “objects per area”, for example, “plants per acre” or “people per square mile”.
* Apply what you know part one:

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| Elu is working on a farm in her town for the summer. She learns that she will be helping to plant 25,000 cucumber seeds on a $\frac{1}{4}$ acre of land. What is the planting density of the cucumber plants? |
| Write out the formula. | $$d=\frac{total \# of objects}{total area}$$ |
| Identify the values needed for the formula.  | The objects in the problem are cucumber seeds and the amount of land in acres is given. $$objects = 25,000 cucumber seeds $$$$area=\frac{1}{4} acres$$ |
| Plug the values into the density formula. | $$d=\frac{25,000}{\frac{1}{4}}$$ |
| Solve for density.  | $$d=100,000 $$ |
| State the answer. | The planting density is 100,000 cucumbers per acre. |

* Apply what you know part two:
	+ Sometimes, the density will be given in a problem, and you will be asked to find the area or number of objects.

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| The current population density of Mathville is 370 people per square mile. If 15,845 people live in Mathville, how many square miles is Mathville? |
| Write out the formula. | $$d=\frac{total \# of objects}{total area}$$ |
| Identify the values needed for the formula. | In this problem, the density and number of objects (people) is given. $$d=370 \frac{people}{mi^{2}}$$$$objects=15,845 people $$$$Area =A $$ |
| Rearrange the formula for the variable you are solving for.  | You are asked to find the total area (A). Let’s rearrange the original formula so that A is on the left side. $$density=\frac{objects}{area}$$$$Area=\frac{objects}{density \left(d\right)}$$ |
| Plug the known values in and solve for the area.  | $$Area=\frac{15,845}{370}=42.82$$ |
| State the answer. | Mathville is approximately 42.82 square miles ($mi^{2}$). |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | The U.S. population was estimated to be 334,233,854 on January 1, 2023, and its area was 3,717,792 square miles. What was the population density of the United States on this day?\_\_\_\_\_ people per square mile | 89.9 |
| P 2 | A horse farm has 60 acres of land and 360 horses. Calculate the density of horses per acre on the farm.\_\_\_\_\_ horses per acre | 6 |
| P 3 | Jada is working on a farm, and she is helping to plant trees on 3 acres of land. Per request, they have to plant 150 trees per acre and finish the work in 90 days. How many trees per day do they have to plant to meet the request?\_\_\_\_\_ trees per day | 5 |
| P 4 | Paris is one of the most densely populated cities in the world with 53,000 people per square mile. If 58,406,000 people live in Paris, how many square miles is Paris?\_\_\_\_\_ mi.2 | 1,102 |
| P 5 | A farm has decided to add chickens to its livestock. Per regulations, the farm cannot have more than 100 chickens per acre. If the area of the farm is 32.4 acres, how many chickens can they have on the farm?\_\_\_\_\_ chickens | 3,240 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | The population of Europe was 748,814,048 people on February 4, 2023, and its area was 3,930,000 square miles. Which of the following correctly calculates the population density of Europe on that day? | 190.54 people per square mile |
| Q 2 | Perhaps you have to plant 10,000 seeds on$\frac{1}{2}$ acre of land. Which of the following correctly calculates the planting density? | 20,000 seeds per acre |
| Q 3 | Martha wants to plant 8,000 seeds. If the planting density is 20,000 seeds per acre, which of the following correctly uses a density calculation to solve for the area of the planting land? | 0.4 acres |
| Q 4 | The population of a country is 52,568,472 people, and its area is 482,594 square miles. Which of the following correctly calculates the population density of the country? | 109 people per square mile |
| Q 5 | John is working on a farm, and he is helping to plant trees on a 4-acre plot of land. Per request, they have to plant 1,200 trees. What is the planting density? | 300 trees per acre |

**Lesson 5 – Density in Three Dimensions**

**Key Words:**

* **density –** a value that describes how compact or concentrated something is; the average number of units per unit of space

**Formulas:**

* 3D Density Formula: $d=\frac{mass}{volume}=\frac{m}{V}$
	+ Mass given density and volume: $m=dV  $
	+ Volume given density and mass: $V=\frac{m}{d}$
* Volume Formulas:
	+ Cylinder: $V=πr^{2}h$
	+ Cone: $V=\frac{1}{3}πr^{2}h$
	+ Cube: $V=lwh $
	+ Prism: $V=A\_{base}⋅h$
	+ Pyramid: $V=\frac{1}{3}lwh or V=\frac{1}{3}ah$
	+ Sphere: $V=\frac{4}{3}πr^{3}$

**Objective 1:** In this section, you will calculate the density of three-dimensional objects.

*Mathematical Practice Standard: Make sense of problems and persevere in solving them.*

**Objective 2:** In this section, you will use the density calculations of three-dimensional objects to solve problems.

*Mathematical Practice Standard: Attend to precision.*

**Big Ideas**:

* Recall that *density* is a value that gives the number of units of something per unit of space. In other words, it describes how compact a three-dimensional object is.
* The *density* of a three-dimensional model can be modeled with this formula:
	+ $d=\frac{mass}{volume}=\frac{m}{V}$
* *Density* of three-dimensional objects is most commonly expressed in cubic units, more specifically, grams per cubic centimeter ($\frac{g}{cm^{3}}$).
* Objects with the same *volume*, but different *mass*, will have different *densities*.
* The *volume* of an object is not always given and you must calculate it based on the dimensions of the object. Recall the [volume formulas](#Bookmark3) for each type of three-dimensional object.
* Apply what you know part one:

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| Auggie just found one of his borther’s small wooden toys under the couch. It is in the shape of a cone and has a *diameter of 4.5 cm* and a *height of 7.5 cm*. He weighs the toy and finds that it is *32 grams*. Calculate the density of the toy.  |
| Write out the density formula.  | $$d=\frac{mass}{volume}=\frac{m}{V}$$ |
| Identify the values needed.  | The mass is given, but the volume needs to be calculated first in order to calculate the density of the toy. $$mass=32 grams $$$$Volume = ? calculate in next step $$ |
| Calculate the volume.  | The toy is in the shape of a cone. Let’s use the volume formula for a cone. $$V=\frac{1}{3}πr^{2}h$$The diameter and height are given. $$d=4.5 cm $$$$h=7.5 cm $$The volume formula for a cone requires the radius, not the diameter. Recall that the radius is half of the diameter. $$r=\frac{d}{2}=\frac{4.5}{2}=2.25 cm$$Plug the values for radius and height in and solve for the volume.  |
| Plug the values for mass and volume into the density formula. Solve for density. |  |
| State the answer. | The density of the toy is $0.80 \frac{g}{cm^{3}}$. |

* Apply what you know part two:
	+ Sometimes, the *density* will be given in a problem, and you will be asked to find the *volume* or *mass* an object.
	+ If you know the *density* and *mass*, you can use this information to calculate the *volume*.
		- $V=\frac{m}{d}$
	+ If you know the *density* and *volume*, you can use this information to calculate the *mass*.
		- $m=dV $

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| Redwood has a *density* of approximately*450 kilograms per cubic meter*. If a redwood tree has a trunk with a height of 60 meters and an average diameter of 3 meters, find the trunk's approximate mass.  |
| Write out the formula needed.  | You are asked to find the mass. $m=dV  $ |
| Identify the values needed. | You are given the density and the dimensions to calculate volume.$$density = 450 \frac{kg}{m^{3}}$$$$volume=? calculate in next step $$ |
| Calculate the volume.  | A tree has a cylindrical shape. The formula for the volume of a cylinder is:$$V=πr^{2}h$$The diameter and height are given. $$d=3 m $$$$h=60 m $$The volume formula for a cylinder requires the radius, not the diameter. Recall that the radius is half of the diameter.$$r=\frac{d}{2}=1.5 m$$Plug the radius and height into the volume formula. $$V=πr^{2}h$$ |
| Plug the values for density and volume into the density formula. Solve for area. |  |
| State the answer. | The mass of the trunk of the redwood tree is approximately 190,800 kilograms. |

**Practice Questions and Answers**

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|  | Question | Answer |
| P 1 | A rectangular prism with a length of 10 cm, width of 8 cm, and height of 16 cm has the mass of 1,248 grams. Find the density of the prism.\_\_\_\_\_ g/cm3 | 0.975 |
| P 2 | The density of silver is 10.49 g/cm3. If a sample of silver has a volume of 0.2 m3, what is the mass of the sample?\_\_\_\_\_g  | 2,098,000 |
| P 3 | A steel cylinder has a base with a radius of 10 cm and a height of 8 cm. If the density of the cylinder is 8,000 kg/m3, find the mass of the cylinder in kilograms. Use $π=3.14$ and round your answer to the nearest hundredth.\_\_\_\_\_kg | 20.1 |
| P 4 | A rectangular prism with a length of 8 centimeters, width of 15 centimeters, and an unknown height has a mass of 1,260 grams and a density of 1.2 g/cm3. Find the height of the prism. \_\_\_\_\_cm | 8.75 |
| P 5 | A jewelry company makes golden circle pendants. Each circle is made of 10 cm3 of gold, and there are 19.32 grams of gold per cubic centimeter. If gold costs $60 per gram, what is the total cost for 12 golden circles?$\_\_\_\_\_ | 139,104 |

**Quick Check Questions and Answers**

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|  | Question | Answer |
| Q 1 | If iron has a density of 7,874 kg/m3, and the mass of an iron block is 16,000 kg, which of the following correctly calculates the volume of the iron block? | 2.03 m3 |
| Q 2 | The density of silver is 10.49 g/cm3, and the density of gold is 19.32 g/cm3. What mass of gold will have the same volume as 12 grams of silver? | 22.10 g |
| Q 3 | A cube of steel has a mass of 1,000 kilograms. What are the dimensions of the cube if the density of steel is 8,000 kg/m3? | 0.5 m |
| Q 4 | Apple juice in a glass has a mass of 0.25 kilograms and a volume of 320 cm3. Which of the following correctly uses this information to calculate the density of the apple juice? | 0.78 g/cm3 |
| Q 5 | The mass of a triangular pyramid is 400 grams and the density of the material from which the pyramid is made is 16 g/cm3. If the height of the pyramid is 5 centimeters, and the base of the pyramid is an equilateral triangle, which of the following correctly calculates its base side length? | 5.89 cm |